Applications
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Platonic Solids
- regular
  - Euler's theorem: for any polyhedron, faces + vertices = edges + 2
  - regular polygon: if all edges of a polygon have the same length and any two edges meeting at a vertex include the same angle
  - regular polyhedron or platonic solid: if all bounding faces of a polyhedron are regular polygons
  - principle of duality or reciprocity: each face of a cube (dodecahedron) is related to a vertex of an octahedron (icosahedron) and vice versa, the centers of the faces of one can be used as the vertex of the other
- there are only five essentially different polyhedrons

<table>
<thead>
<tr>
<th>platonic solid</th>
<th>faces</th>
<th>edges</th>
<th>vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>tetrahedron</td>
<td>4</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>cube (hexahedron)</td>
<td>6</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>octahedron</td>
<td>8</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>dodecahedron</td>
<td>20</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>icosahedron</td>
<td>20</td>
<td>30</td>
<td>12</td>
</tr>
</tbody>
</table>

- tetrahedron
  - using the diagonals of the six faces of a cube (with edges of length 2 and with its center as the origin) as the edges of the tetrahedron
- cube (hexahedron) and octahedron
- dodecahedron and icosahedron
  - in the icosahedron, there are 2 horizontal pentagons with their vertices on (horizontal) circles with radius 1; these 2 horizontal pentagons lie a distance 1 apart, while the 'north pole' and the 'south pole' lie a distance $5^{1/2}$ apart; the coordinate of all 12 vertices of an icosahedron are:

<table>
<thead>
<tr>
<th>$i$</th>
<th>$x_i$</th>
<th>$y_i$</th>
<th>$z_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$5^{1/2}$</td>
<td>0</td>
</tr>
<tr>
<td>2, 3, 4, 5, 6</td>
<td>$\cos((i-2)*72^\circ)$</td>
<td>0.5</td>
<td>$\sin((i-2)*72^\circ)$</td>
</tr>
<tr>
<td>7, 8, 9, 10, 11</td>
<td>$\cos(36^\circ+(i-7)*72^\circ)$</td>
<td>-0.5</td>
<td>$\sin(36^\circ+(i-7)*72^\circ)$</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>$-5^{1/2}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Sphere Representation
- globe model
  - with north and poles at the top and the bottom, horizontal circles called lines of latitude, and circles called lines of longitude in vertical planes through the poles; a single integer $n$, indicating both the number of horizontal slices and half the number of lines of longitude, i.e., angles of $180^\circ/n$ play an essential role
  - 2 drawbacks
    - the faces are unequal in size and have different shapes: except for the triangles at the poles, each face is a trapezium, whose size depends on its distance from its nearest pole
    - these spheres have an 'anisotropic' appearance, e.g., a view from above looks much different from a view from the front
  - spheres based on an icosahedron
    - divide each triangle face of an icosahedron into 4 triangles by using the midpoints of all 3 edges of the original triangle and raise the midpoints up to make on the sphere (with origin as center and radius of 1); we obtain a polygon with 80 faces (not equilateral triangles though), 42 vertices, and 80 + 42 - 2 = 120 edges; we can apply this recursively
    - a more general solution by using a input file contains a convex polyhedron that has only triangular faces and whose center is the origin of the coordinate system; the output file is a polyhedron that has 4 times as many (triangular) faces as the input one, and whose vertices lie on a sphere with center at origin and radius 1

Torus
- a torus has $n$ small vertical circles (with radius $r = 1$), and a large horizontal circle (with radius $R > r$ to prevent smaller circles intersect each other) containing the centers of the $n$ smaller circles
- a parametric representation of the large circle is $x = R \cos \alpha$, $y = R \sin \alpha$, $z = 0$; if we take $\alpha = 0$, we obtain the center of the small vertical circle with the parametric representation $x = R + r \cos \beta$, $y = 0$, $z = r \sin \beta$; this small circle belong to $i = 0$; by rotating it about the $z$-axis through angles $\alpha = i \delta$, where $i = 1, 2, .., n-1$ and $\delta = 2\pi/n$, we obtain the remaining $n-1$ small circles; as for the vertex numbers of the torus, we select $n$ points on the first small circle (corresponding to $i = 0$) and assign the integer $1, 2, .., n$ to them; the point obtained by using parameter $\beta = j \delta$ is assigned vertex number $j+1$ ($j = 0, 1, ..., n-1$); the next $n$ vertices, numbered $n+1$, $n+2$, ..., $2n$, lie on the neighboring circle, corresponding to $i = 1$, and so on; in general, we use the $n^2$ vertex numbers $i^*n+j+1$ ($i = 0, 1, ..., n-1; j = 0, 1, ..., n-1$)

Beams in a Spiral
- a spiral built from $n$ horizontal beams with length $2a$ ($a \geq 0.5$), width 1 and height 1; the bottom of the lowest beam lies in the $xz$-plane; this beam is parallel to the $z$-axis and its maximum $x$-coordinate is equal to $a$; each next beam can be obtained by lifting the previous one a distance 1, and by rotating it about the $y$-axis though a given angle $\alpha$