1. Give proofs of the following asymptotic bounds:
   a) \( \log(n!) = \Theta(n \log n) \)
   b) \( (\log n)^2 = o(n) \) [small oh, not big oh]

2. Consider a binary heap, with the maximal element at the root, represented by an array \( A[1, \ldots n] \). Suppose you need to change the value of its \( i \)-th element. Give an efficient algorithm that sets \( A[i] = x \) and then updates the heap structure appropriately (so that the heap property is preserved).

3. The classical Traveling Salesman Problem has a weighted graph as input and requires to find a minimum-weight cycle that includes all vertices of the graph (a minimal weight tour). We consider a modified version that requires to find a minimum-weight path through all vertices; that is, the salesman has to visit all vertices exactly once, but without returning to the initial version. The original problem is known to be NP-complete. Show that the modified problem also is NP-complete.

4. A directed graph \( G = (V,E) \) is weakly connected if for every pair of vertices \( u \) and \( v \) there exists either a path from \( u \) to \( v \) or a path from \( v \) to \( u \). Given \( G \) in the form of an adjacency list, give an \( O(|V| + |E|) \) algorithm that determines whether a given graph is weakly connected.

5. Suppose that you are using a programming language that allows only integer numbers and supports only three operations on them: addition, subtraction, and multiplication. The running time of these operations is constant. Write an efficient algorithm \( \text{Divide}(n,m) \) that computes \([n/m]\) where \( n \) and \( m \) are arbitrary positive integers and give the complexity of your algorithm. You need to do better than \( \Theta(n/m) \).