The square $G^2$ of a directed graph $G = (V, E)$ is a directed graph with the same set of vertices, but the edge set contains $(u, v)$ if and only if there is another vertex $w$ such that $(u, w), (w, v) \in E$. For each popular representation of a matrix, give efficient algorithms to calculate $G^2$.

An Euler tour of a strongly connected, directed graph $G = (V, E)$ is a numbering $e_i, i = 1, \ldots, n$ of all the edges so that $e_i$ ends in a vertex $v$ and $e_{i+1}$ starts in the same vertex $v$. In addition, the last edge $e_n$ ends in the starting vertex of $e_1$. Show that $G$ has an Euler tour if and only if every vertex has in-degree $=$ out-degree. Describe an $O(E)$ algorithm that finds a Euler tour of $G$ if such a tour exists. How about Euler paths?