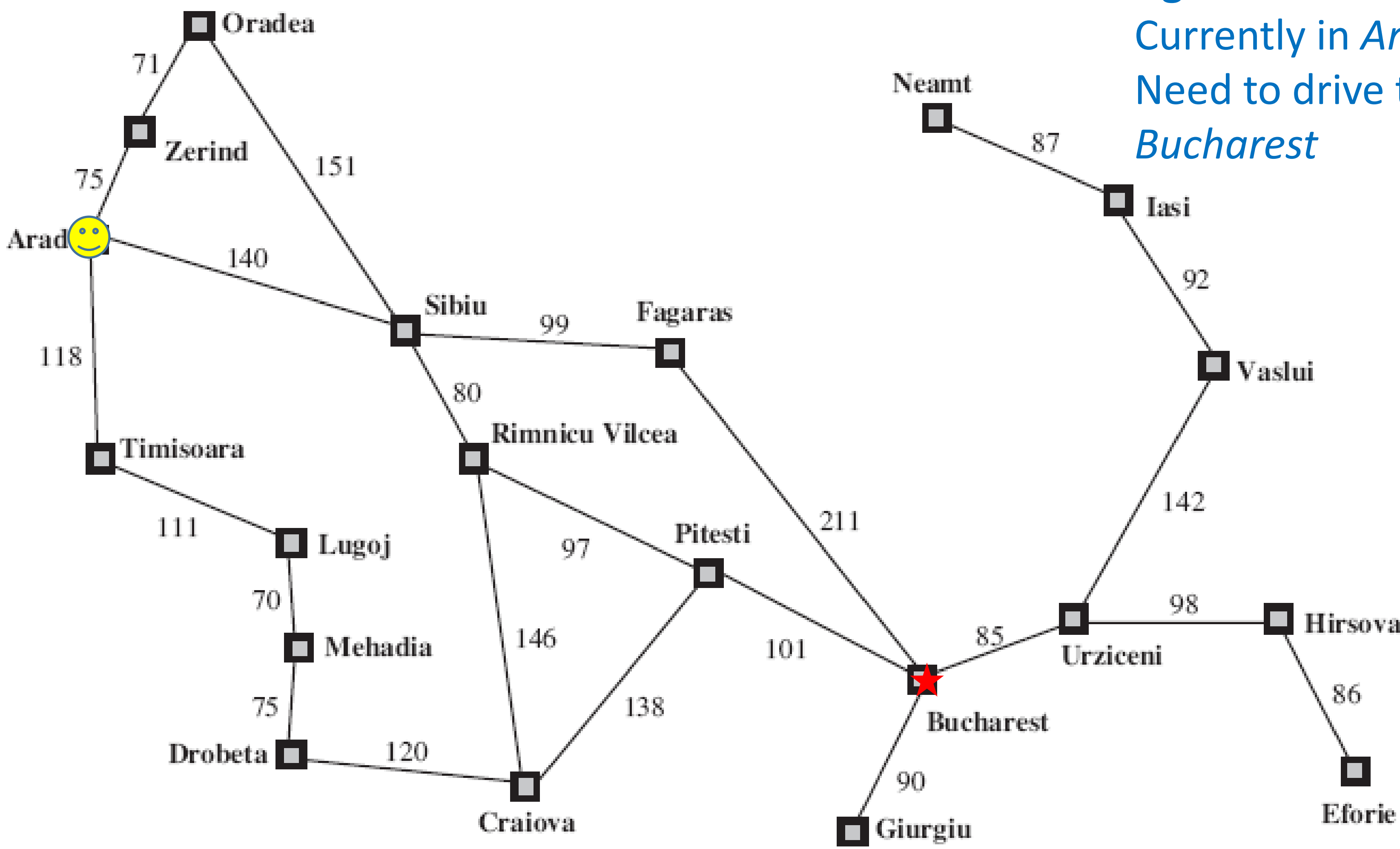
Solving Problems by Searching

CSEN266

Artificial Intelligence

Navigation in Romania



Agent:

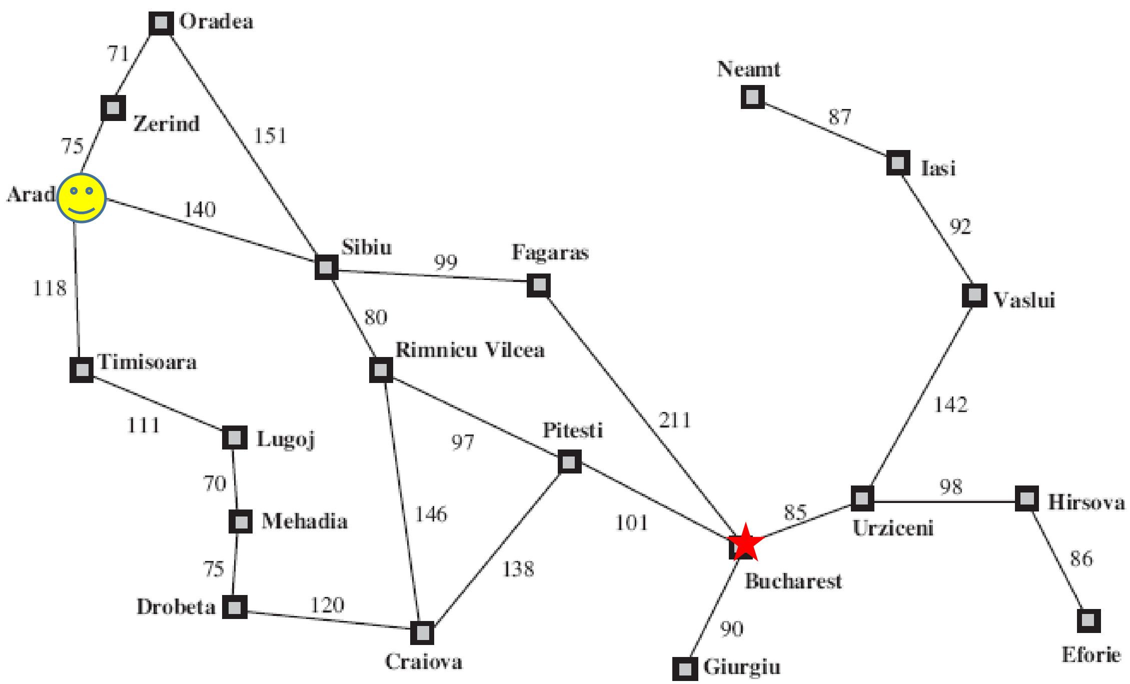
Currently in *Arad*;

Need to drive to

Bucharest

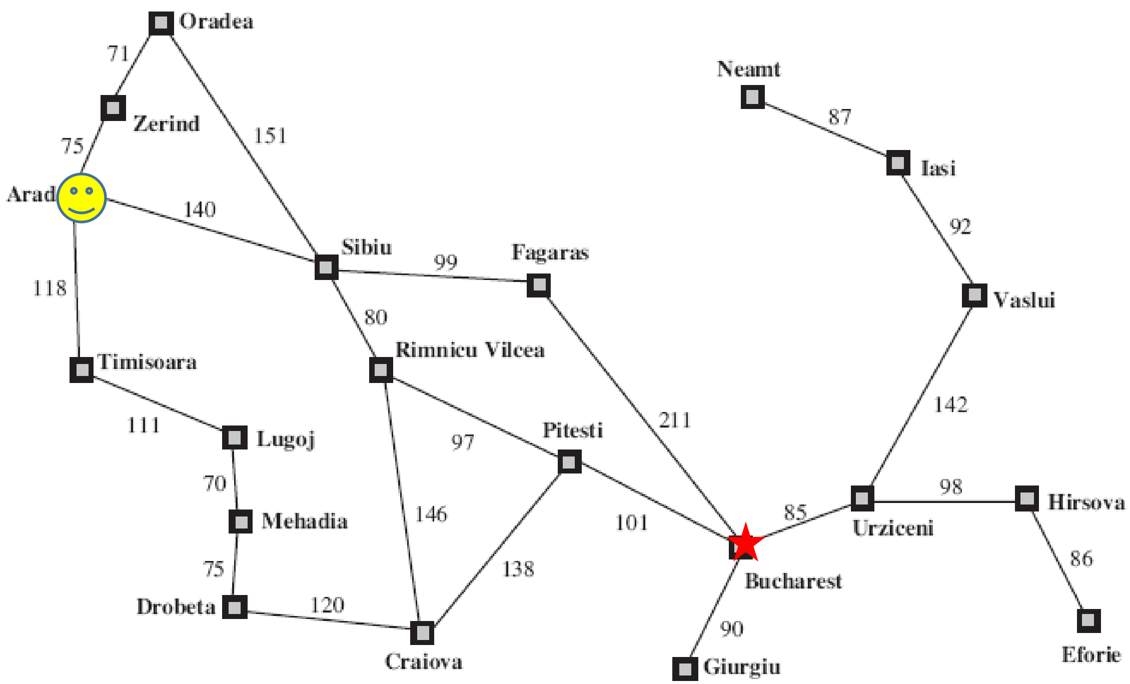
The **goal** of the problem?

- Get to Bucharest from Arad



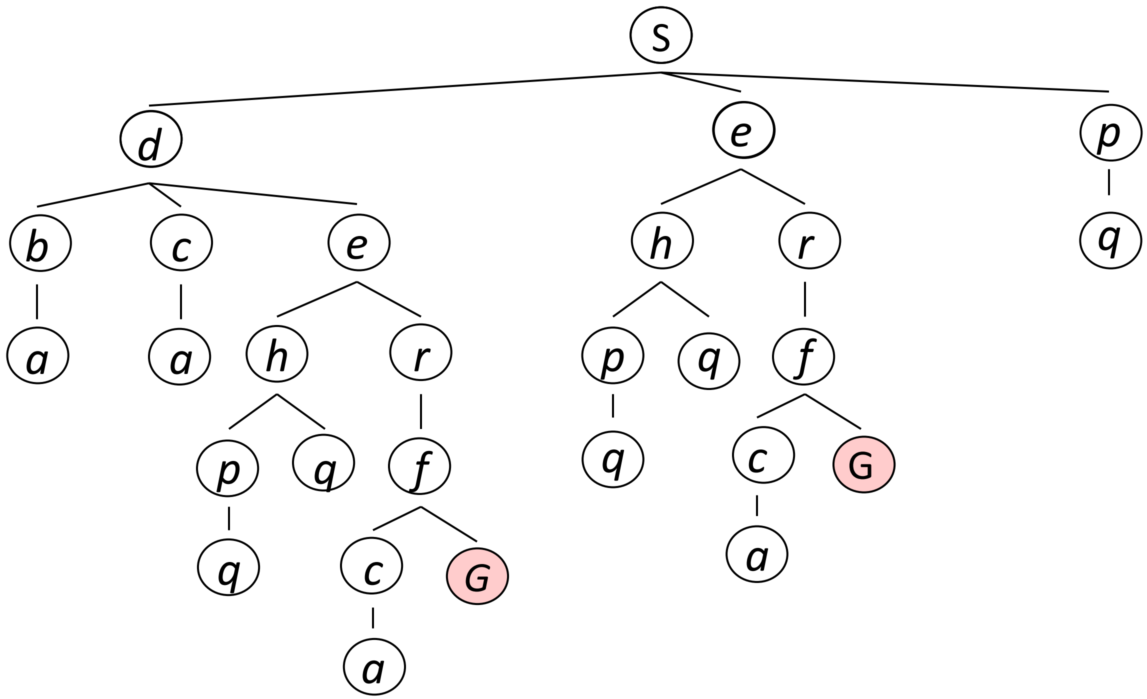
How to achieve the goal?

- Where to go from Arad? Sibiu, Timisoara, or Zerind?
- What actions to take? ⇒ Making decisions



Search Problem

- **Search:** look for a sequence of actions that reaches the goal



An Agent Wants to Do a Task

- **Search Phase**: the process of looking for the action sequence that achieves the goal

  - **Search Algorithm:** takes a problem as input and returns a solution

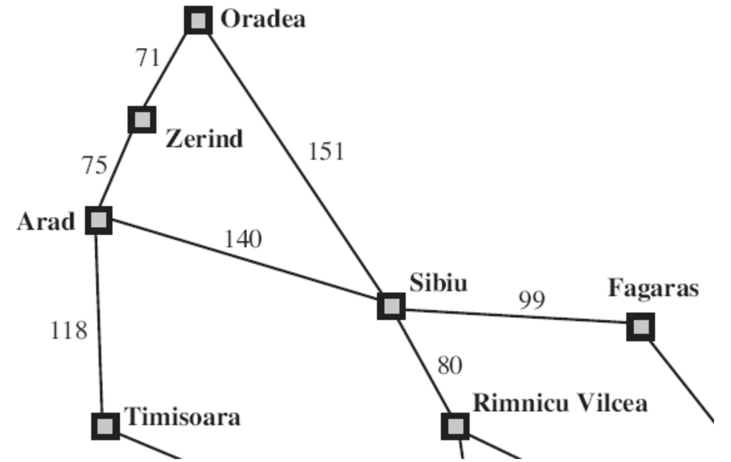
    ⬇

  - **Solution:** an action sequence that leads from the initial state to a goal state

- **Execution Phase:** the agent will carry out the actions recommended by the solution

Define a Search Problem

- Five components
  1. **States**
     e.g. *In(Arad), In(Sibiu), In(Fagaras)*
     Initial State: *In*(*Arad*)
     Goal State: *In*(*Bucharest*)



  2. **Actions** (available to the agent)
     - Given a state *s*, **Actions(*s*)** returns the set of actions that can be executed in *s*
     - **Actions(*In*(*Arad*)): {*Go*(*Sibiu*), *Go*(*Timisoara*), *Go*(*Zerind*)}**

Define a Search Problem

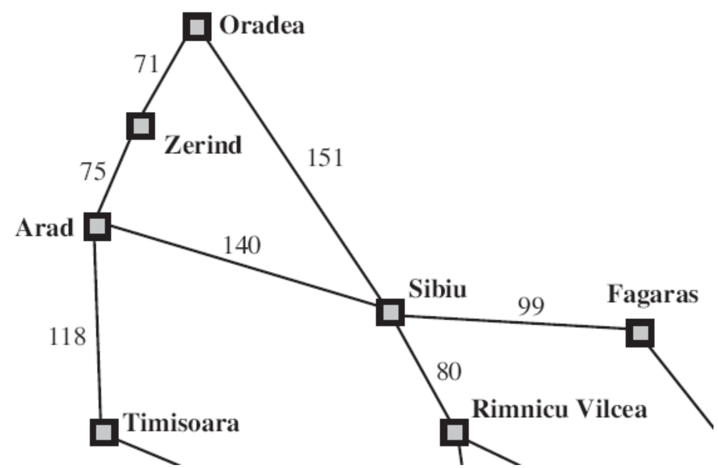
- Five components

3. Transition Model

A description of what each action does

- **Result(*s*,*a*)**: returns the state that results from doing action *a* in state *s*

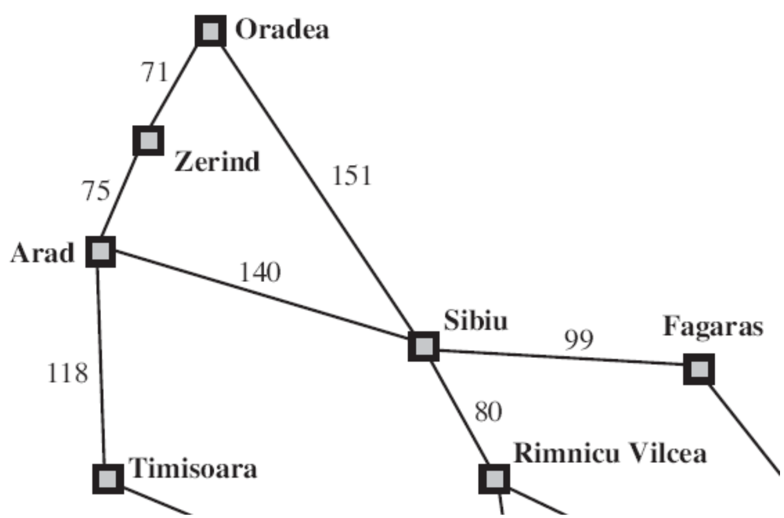
- **Result(In(Arad), Go(zerind)) = In(Zerind)**



Define a Search Problem

- Five components

4. **Goal test:** a function that verifies whether a given state is a goal state
   - e.g. In the Romania Navigation problem: the goal state is the singleton set {*In*(*Bucharest*)}
5. **Path cost function:** assigns a numeric cost to each path
   - e.g. Romania: the cost of a path can be its length in kilometers
   - step cost ***c*(*s*,*a*,*s'*)**: the cost of taking action ***a*** in state ***s*** to reach state ***s'***

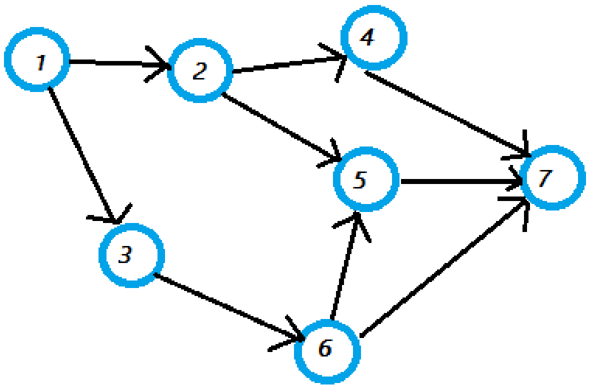


Define a Search Problem

- Additional Concepts

  The *State Space:* the set of all states reachable from the initial state by any sequence of actions.

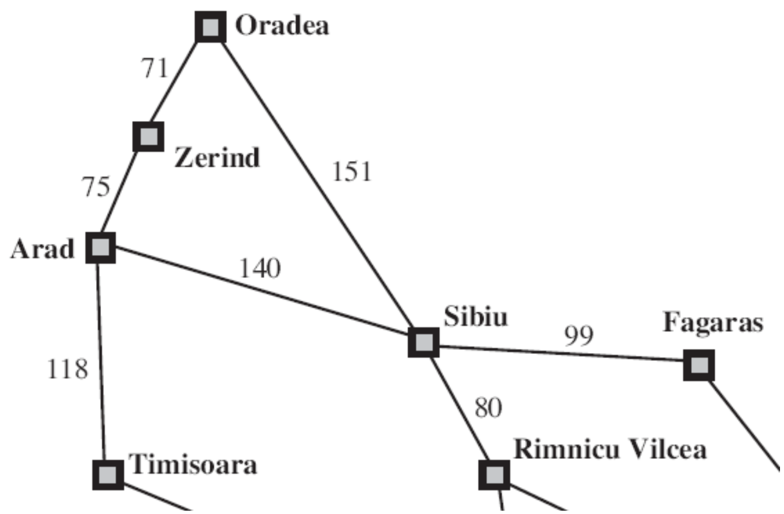
  - A directed graph (e.g. the map of Romania)
    - Nodes: the states
    - Links: actions



  - A *path* in the state space: a sequence of states connected by a sequence of actions

The Solution of a Search Problem

- An action sequence that leads from the initial state to a goal state.
- The quality of the solution: measured by the path cost function
- Optimal solution: lowest path cost



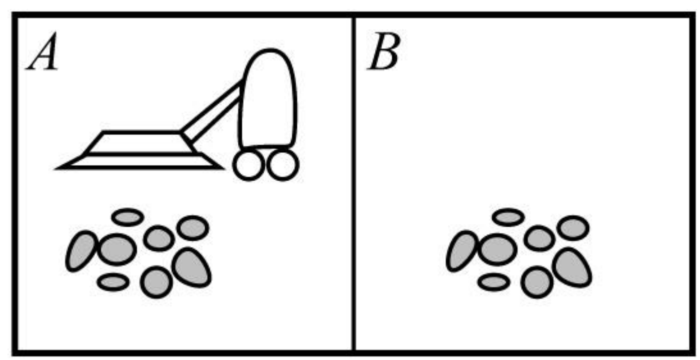
Vacuum World

- States
  - 2 agent locations (squares), each location might or might not contain dirt

    e.g. [‘Dirty’, ‘Dirty’, ‘A’] represents: square A is dirty, square B is dirty, the robot is currently in square A.

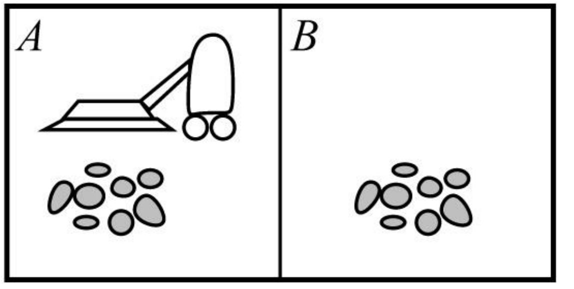
  - $2 \times 2^2 = 8$ states
  - If n locations, then how many states?

    $n \times 2^n$ states



Vacuum World

- Initial state
  - Any state can be designated as the initial state
- Actions
  - Left, Right, Suck
  - Larger environments: Up, Down



- Transition Model
  - e.g. s= [‘Dirty’, ‘Dirty’, ‘A’], a=‘Suck’

    s’ = Result(s, a) = [‘Clean’, ‘Dirty’, ‘A’]

Vacuum World

The complete state space (and transition model)

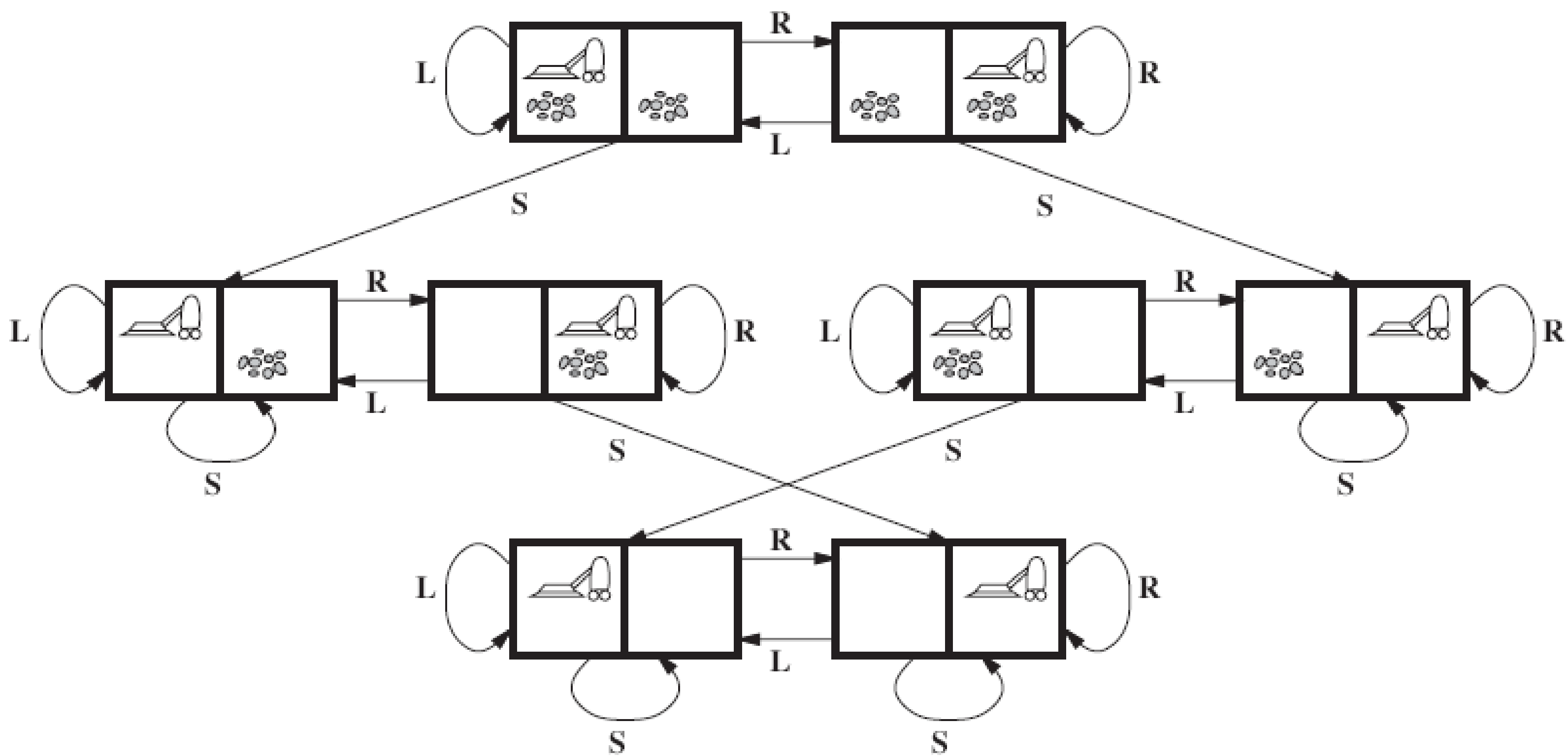


Figure 3.3 FILES: figures/vacuum2-state-space.eps (Tue Nov 3 16:24:01 2009). The state space for the vacuum world. Links denote actions: L = *Left*, R = *Right*, S = *Suck*.

Vacuum World

- Goal Test
  - Check whether all squares are clean
- Path cost
  - Each step costs 1 (i.e. 1 per action)
  - The path cost is the number of steps (actions) in the path

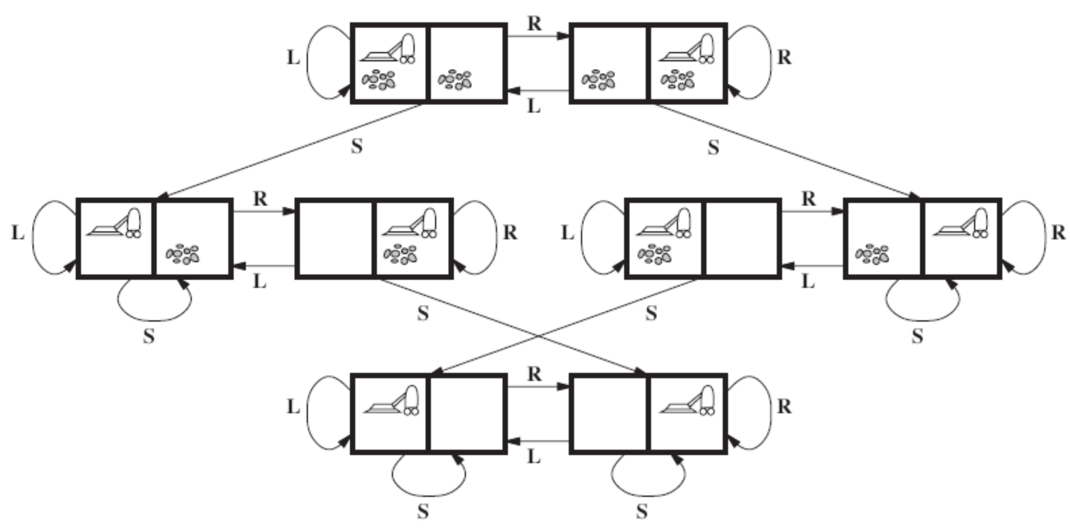
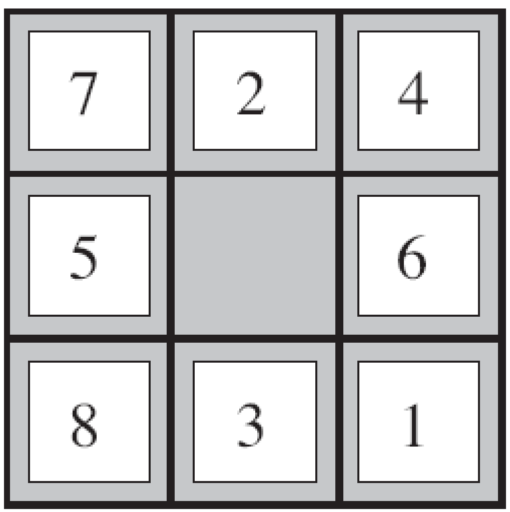


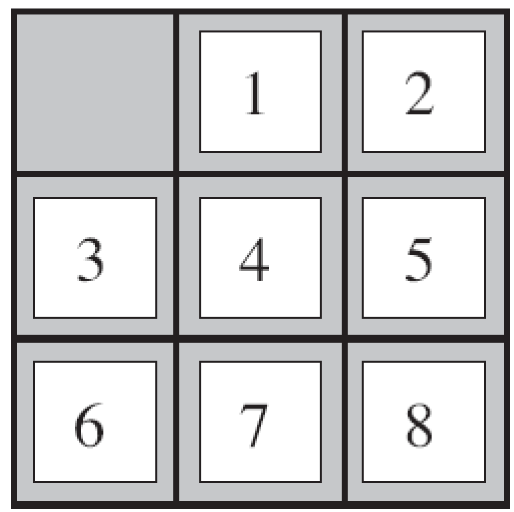
Figure 3.3 FILES: figures/vacuum2-state-space.eps (Tue Nov 3 16:24:01 2009). The state space for the vacuum world. Links denote actions: L = *Left*, R = *Right*, S = *Suck*.

The 8-puzzle

- A tile adjacent to the blank space can slide into the space
- Objective: reach a specified goal state



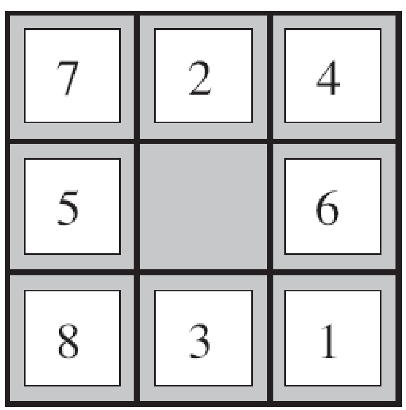
Start State



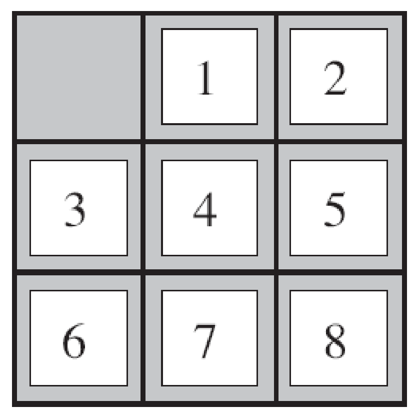
Goal State

The 8-puzzle

- States?
  - any location combination of 8 tiles and the blank
- Initial state?
  - given
- Actions?
  - Movements of the blank space: left, right, up, down, or a subset of these



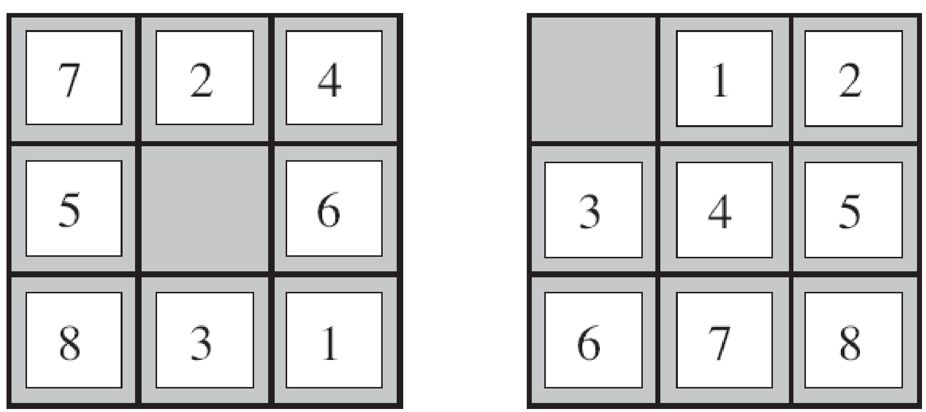
Start State



Goal State

The 8-puzzle

- Transition model?
  - given a location combination and a movement, this returns the resulting location combination
- Goal test?
  - check whether the state matches the goal configuration
- Path cost?
  - (each step costs 1) the number of steps in the path



Start State Goal State

- Donald Knuth (1964): starting with the number 4, a sequence of factorial, square root, and floor operations will reach any desired positive integer.

$$\left\lfloor \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}}} \right\rfloor = 5$$

- States?
  - Positive numbers
- Initial state?
  - 4
- Actions?
  - Apply factorial, square root, or floor operation

- Donald Knuth (1964): starting with the number 4, a sequence of factorial, square root, and floor operations will reach any desired positive integer.

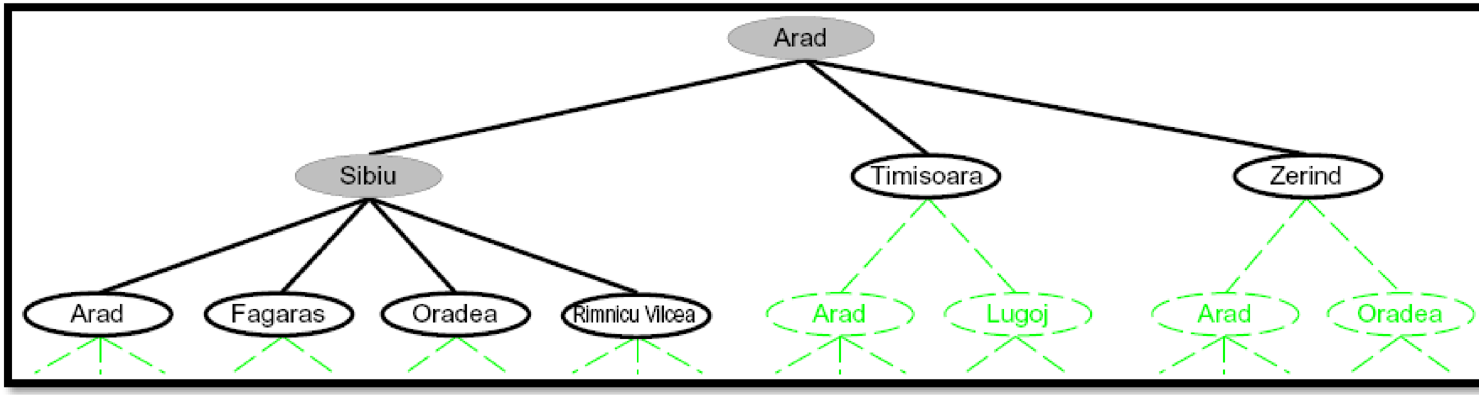
$$\left\lfloor \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}}} \right\rfloor = 5$$

- Transition model?
  - Definition of these math operations
- Goal test?
  - State is the desired positive integer

- Infinite state space

Search Algorithms

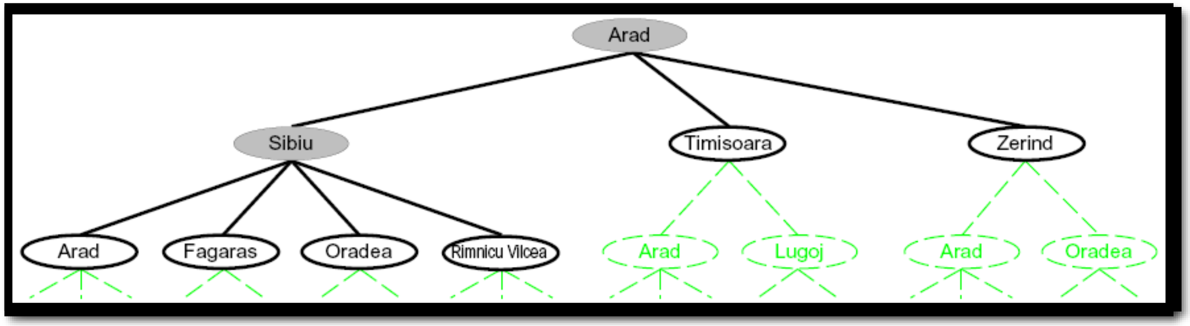
- Look for a sequence of actions that achieve the goal
- **Search Tree**

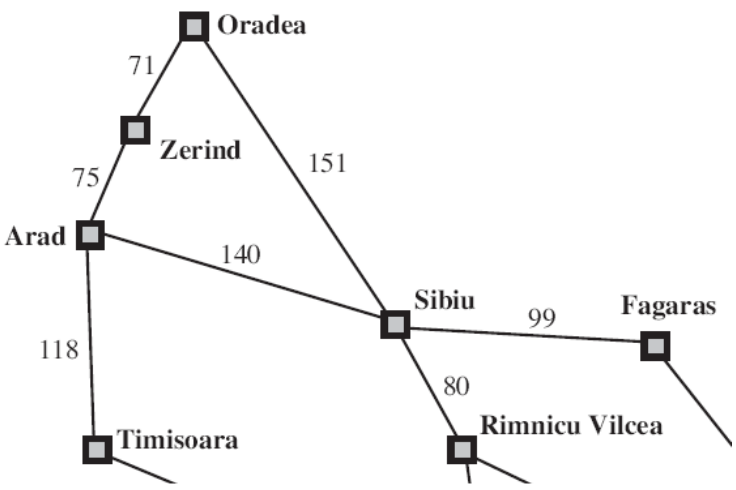


  - **Nodes:** states in the state space
    - Parent node, child nodes
    - Leaf node: a node with no children
  - **Root Node:** initial state
    - In(Arad)
  - **Branches:** actions

Search Algorithms

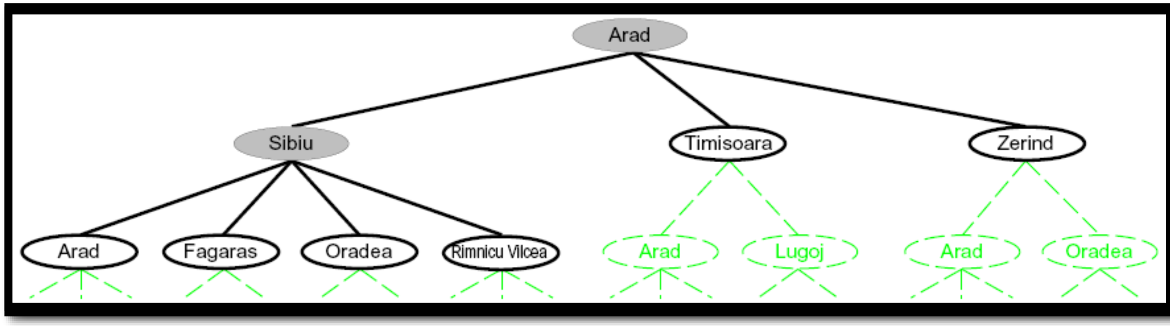
- Expand a node and generate a new set of nodes
- How to choose which node to expand?

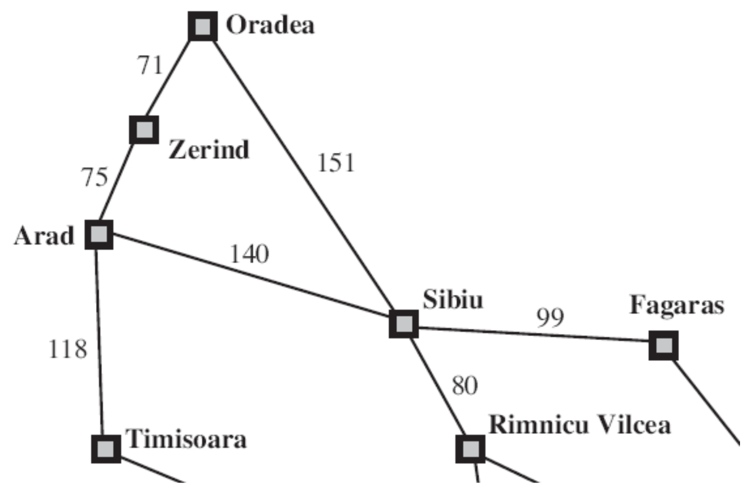




Search Algorithms

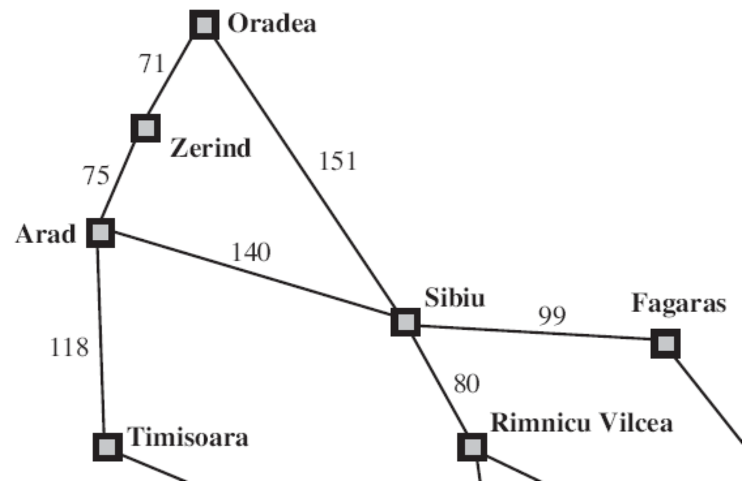
- Frontier nodes: the candidate nodes to be expanded

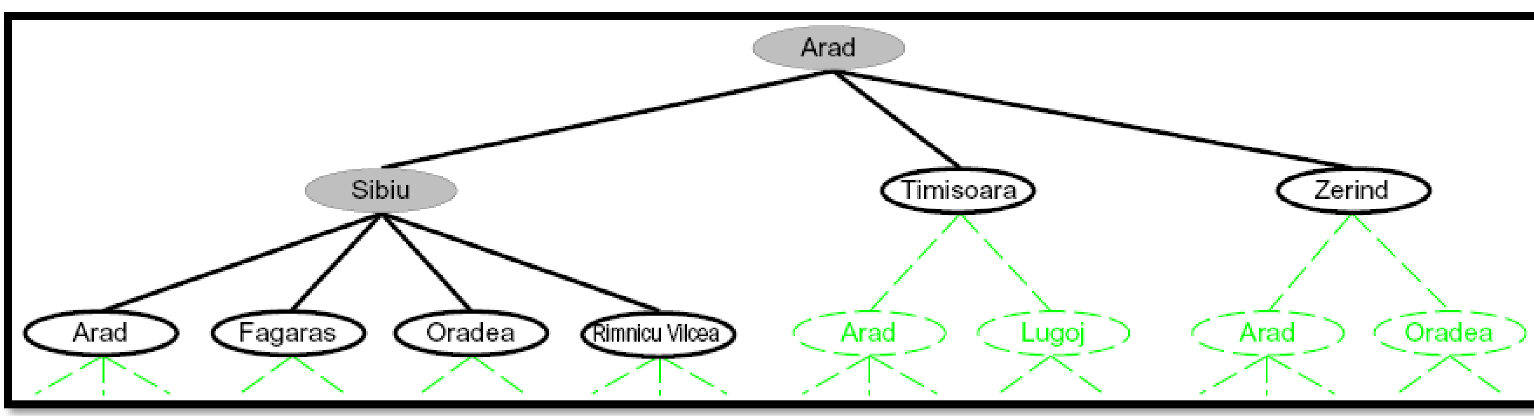




Tree-Search Scheme

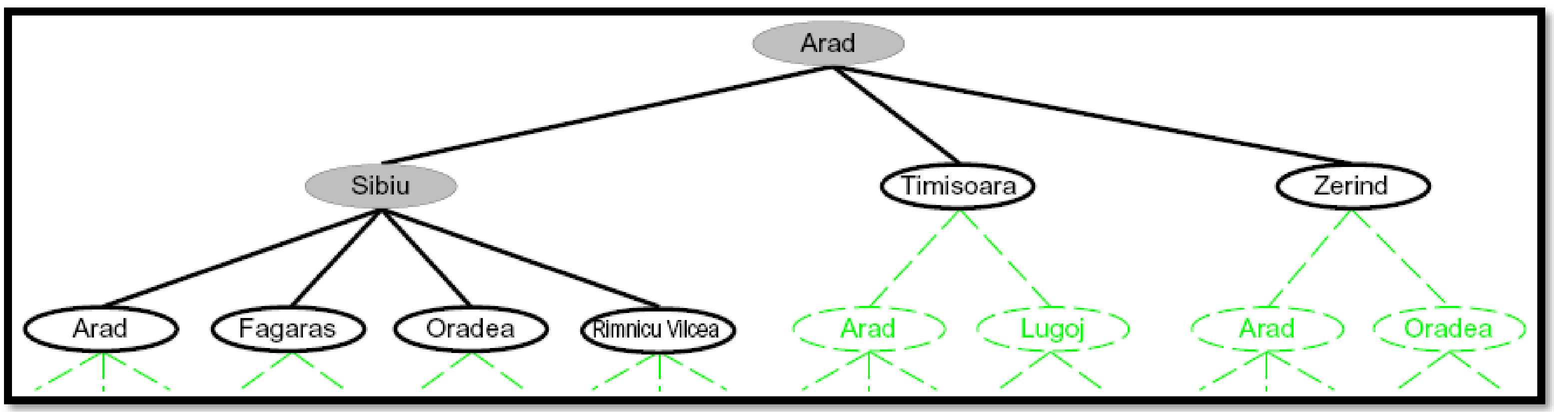
- The same state can be visited repeatedly
  - Arad-Sibiu-Arad
  - Loopy path
    - The complete search tree for Romania is *infinite*
    - Can cause certain algorithms to fail





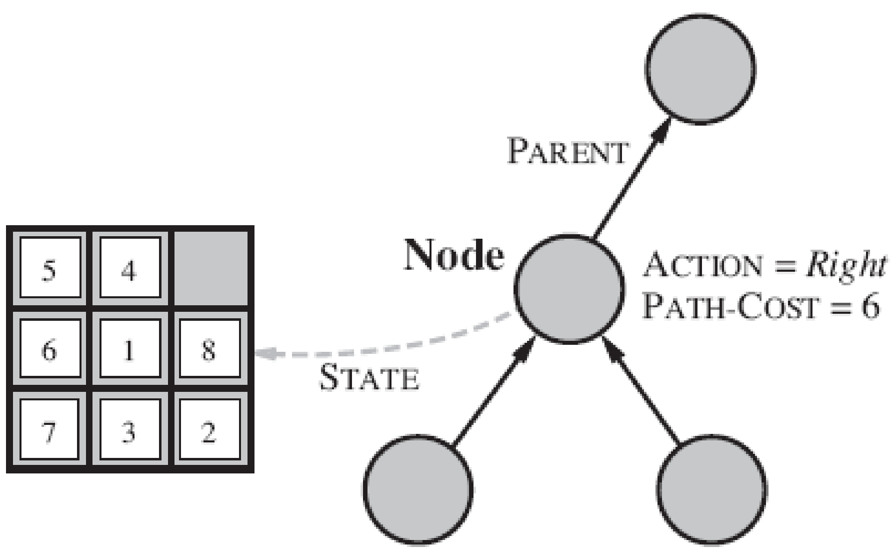
Graph-Search Scheme

- Never expands the same state twice



Data structure for node n on the search tree

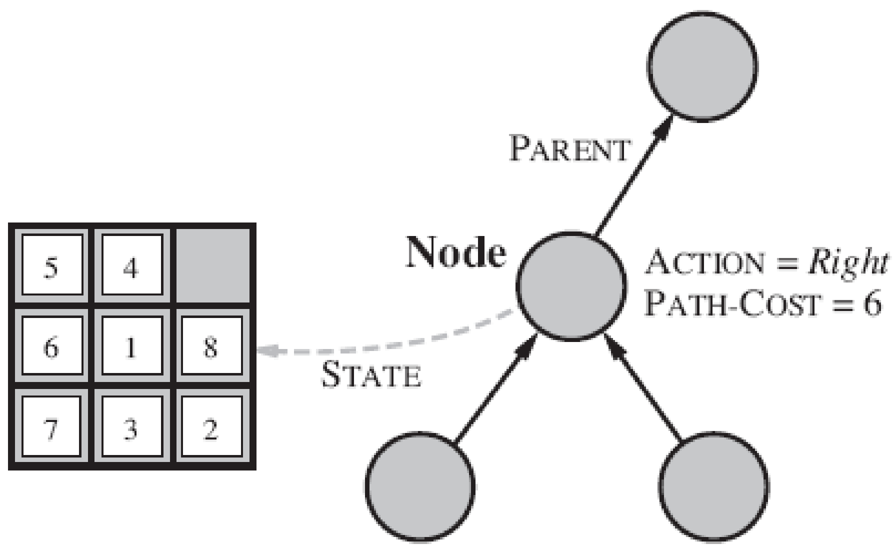
- Data structure: keeps track of the search tree
- n.State: the state in the state space to which the node corresponds.
- n.Parent: the node in the search tree that generated this node.
  - Arrows point from child to parent



Data structure for node n on the search tree

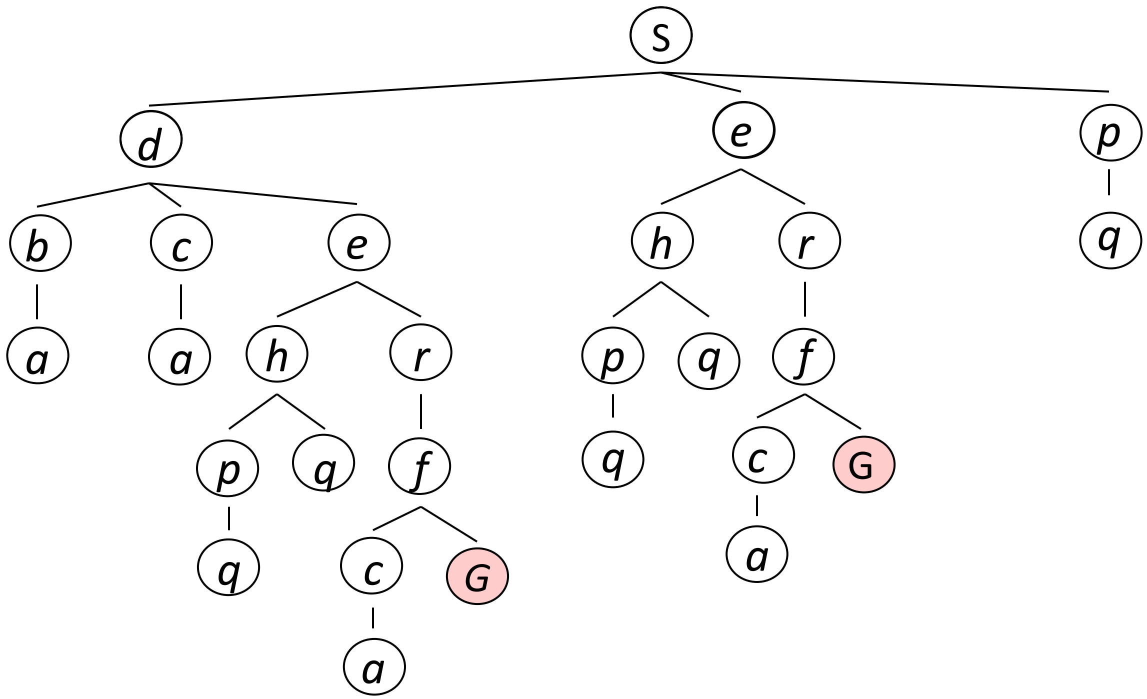
- n.Action: the action that was applied to the parent to generate the node.
  - e.g. n.Action: movement of the blank tile

    Draw the state of the parent node?
- n.Path-Cost $g(n)$: the cost of the path from the initial node to node n, as indicated by the parent pointers.



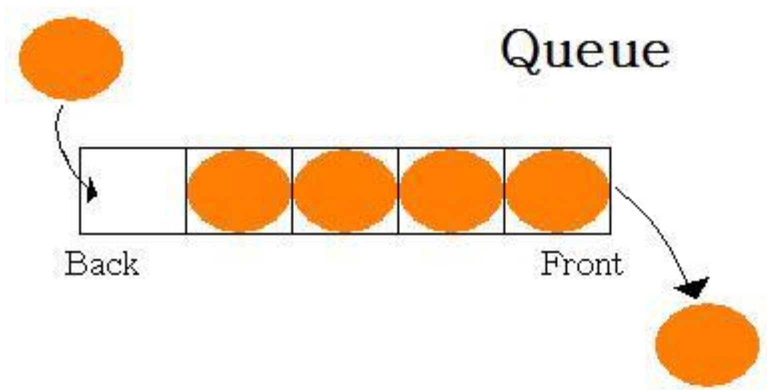
Data structure for node n on the search tree

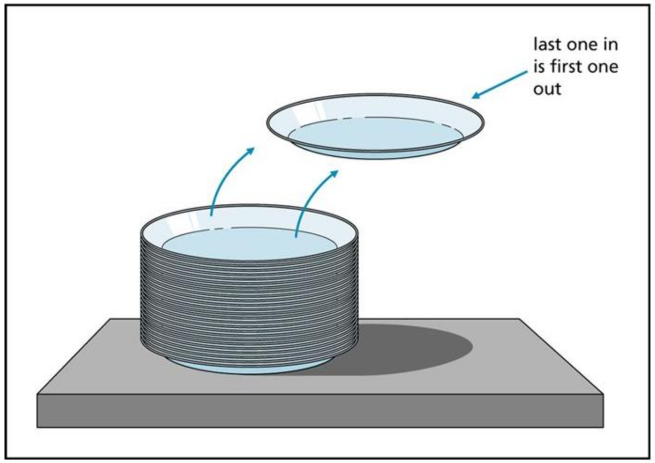
- **Illustrate:** n.State, n.Parent, n.Action, n.Path-Cost=$g(n)$



Frontier Set

- The set of Frontier nodes
  - a FIFO queue, or
  - a LIFO stack, or
  - a Priority queue





Performance Measurement

- Completeness: Is the search algorithm guaranteed to find a solution when there is one?

- Optimality: Does the strategy find the optimal solution (w.r.t some performance measure)?
  - e.g. The path cost of the solution found (such as the total length of the path in kilometers)

Performance Measurement

- Time complexity: How fast does the algorithm find a solution?
  - Measured in terms of the number of nodes expanded/visited/explored during the search process

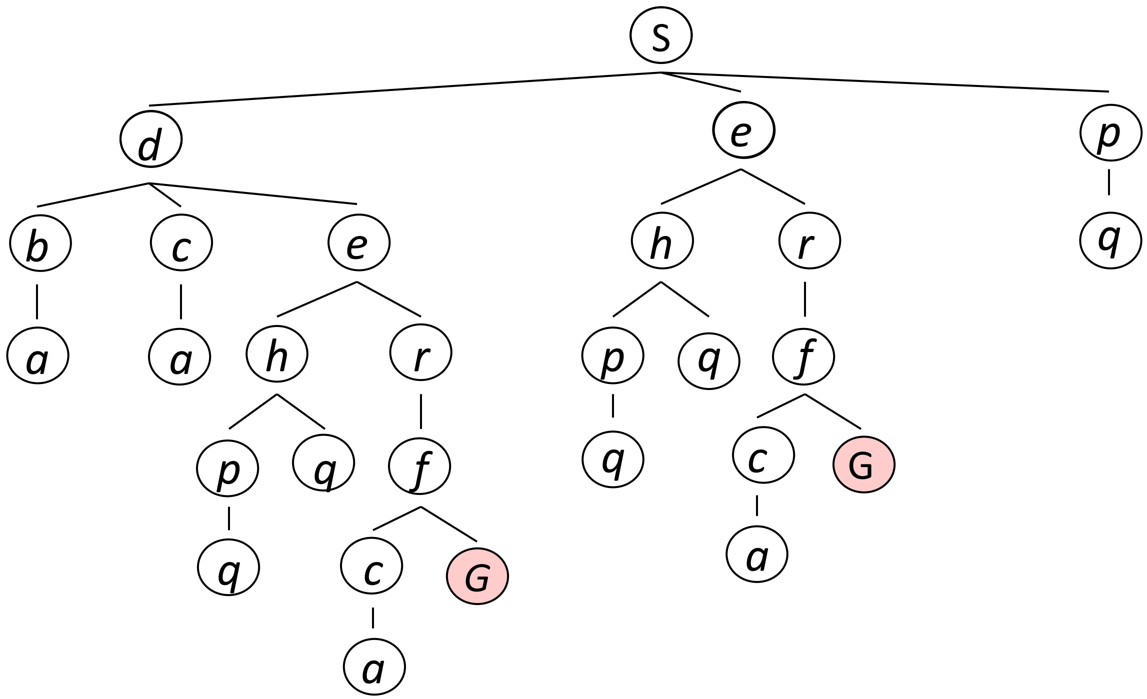
- Space complexity: How much memory is needed to perform the search?
  - Measured in terms of the maximum number of nodes stored in memory during the search process

Parameters

- b - Branching factor
  - Maximum number of successors of any node
- d - The *depth* of the shallowest goal node (shallowest solution)
  - i.e. the number of steps along the path from the root
- m – the *maximum length* of any path in the state space
  - For tree search
    - m can be much larger than d
    - m is infinite if the tree is unbounded

Parameters

- What are b, d, and m for this search tree?

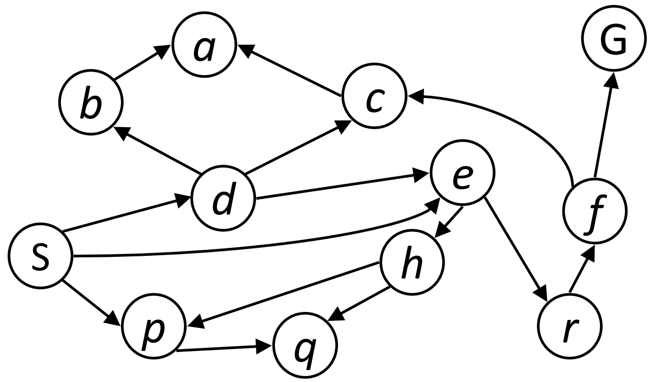


Uninformed Search

- Also called: Blind Search
- The search strategy
  - does not know which non-goal states are better than other non-goal states
  - can only
    - Generate successors
    - Distinguish a goal state from a non-goal state

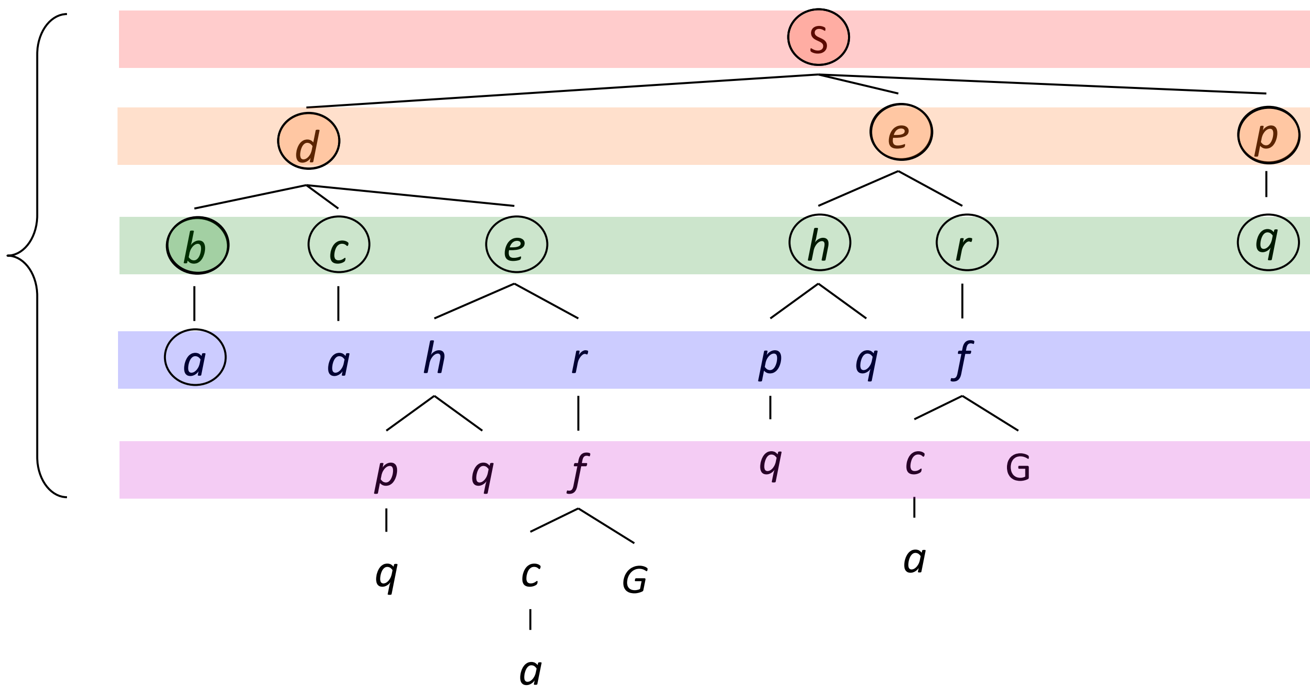
- Different uninformed search strategies
  - distinguished by the **order** in which nodes are expanded

Breadth-First Search



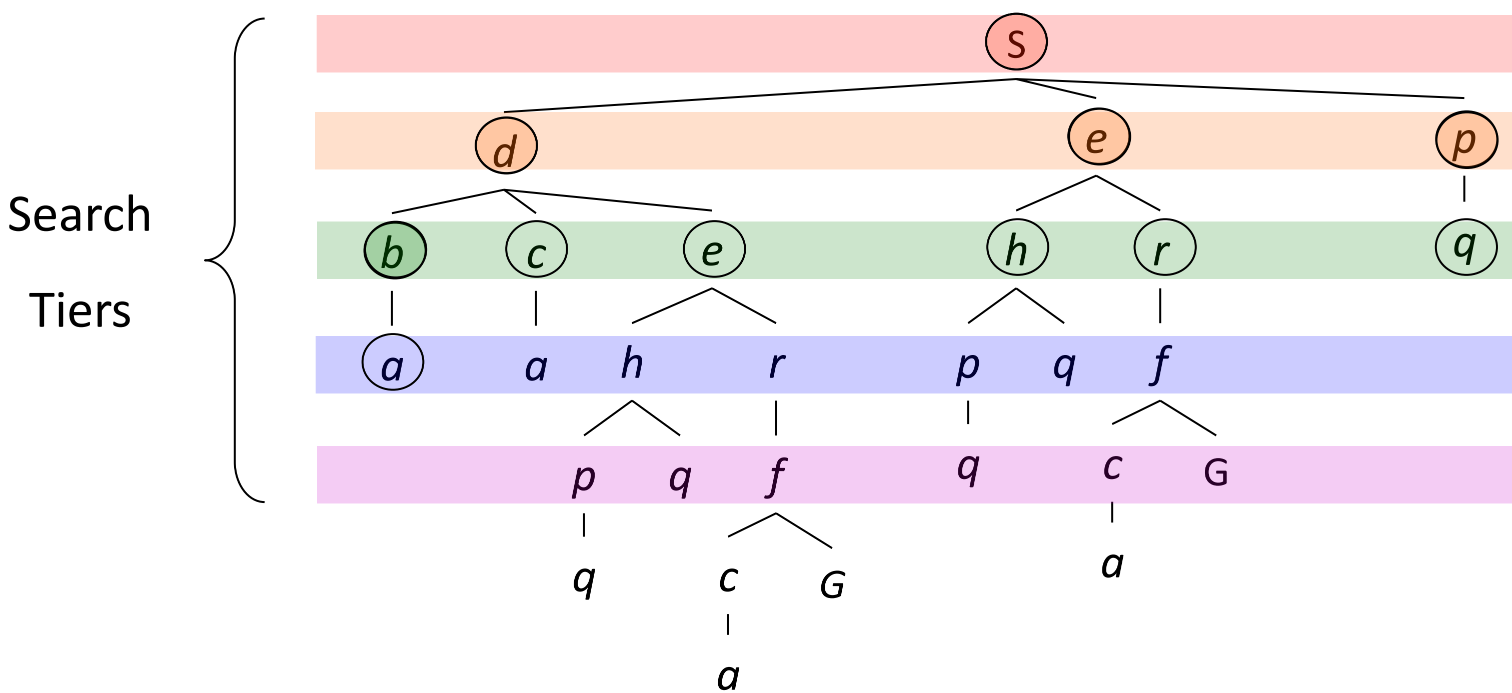
Initial State: S
Goal State: G
Explored: color circled
Frontier: white circled
Unexplored: uncircled

Search Tiers



Breadth-First Search

- Shallower nodes are expanded before deeper nodes
- Achieved by using a FIFO queue for the frontier nodes



Breadth-First Search

- Optimality?
  - In terms of solution path cost
  - Optimal in the sense that it always finds the shallowest goal node

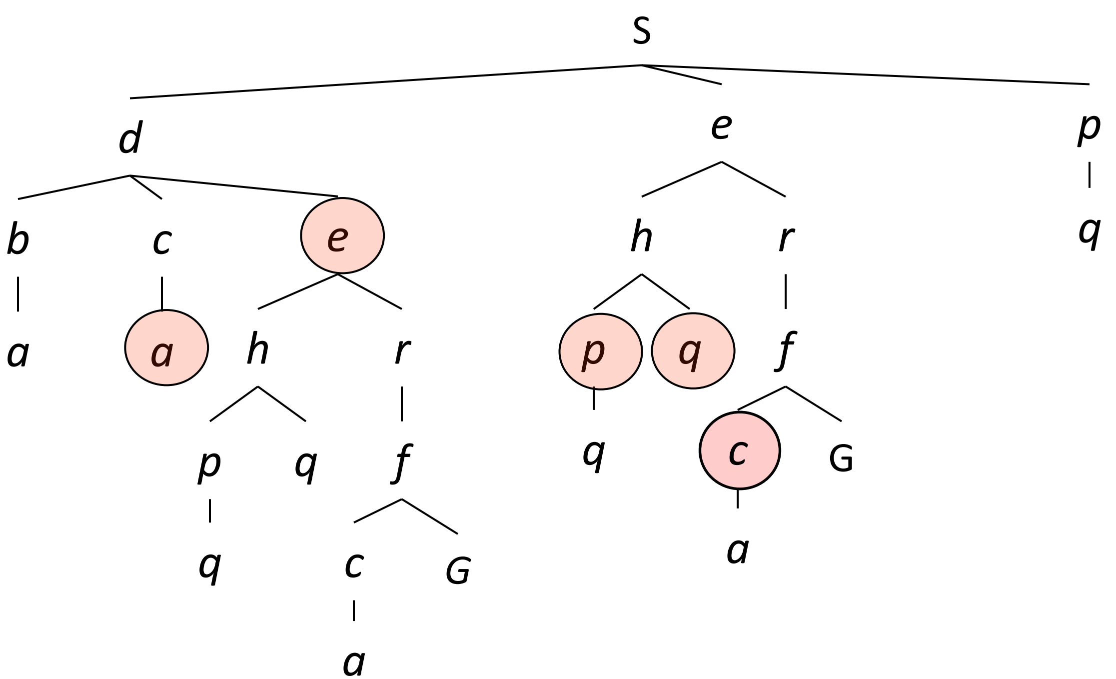
- Complete?
  - Yes if the branching factor b is finite

Graph-Search Scheme

- Idea: never visit the same state twice
- How to implement:
  - Construct a search tree
  - Keep a set of visited states
  - Expand the search tree node-by-node as in a tree search strategy, but…
  - Before expanding a node, check whether its state has been visited before
    - If yes, skip the node;
    - If no
      - add its state to the set of visited states
      - expand the node and generate the successors

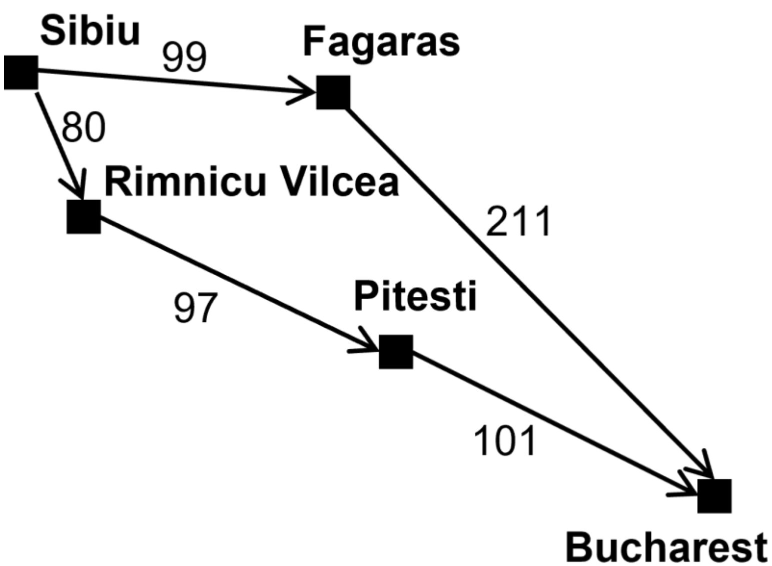
Graph-Search Scheme

- Idea: never visit the same state twice
- Example: in the following breadth-first graph search, we do not expand the circled nodes



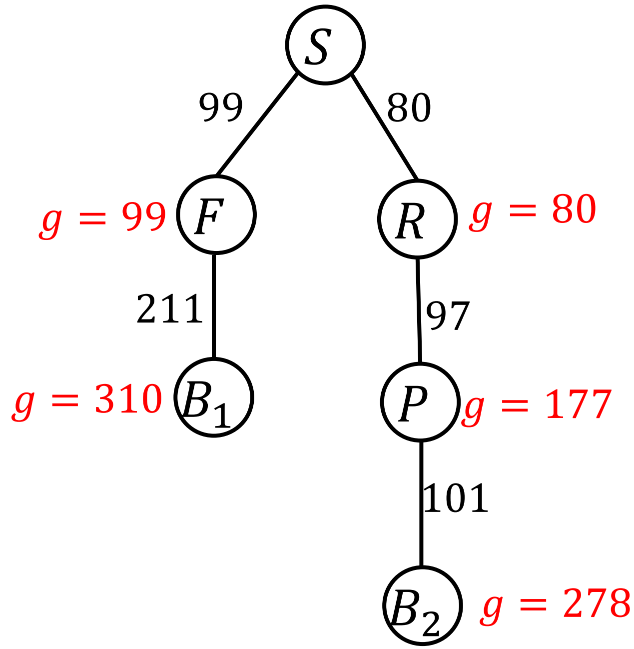
Uniform-cost Search

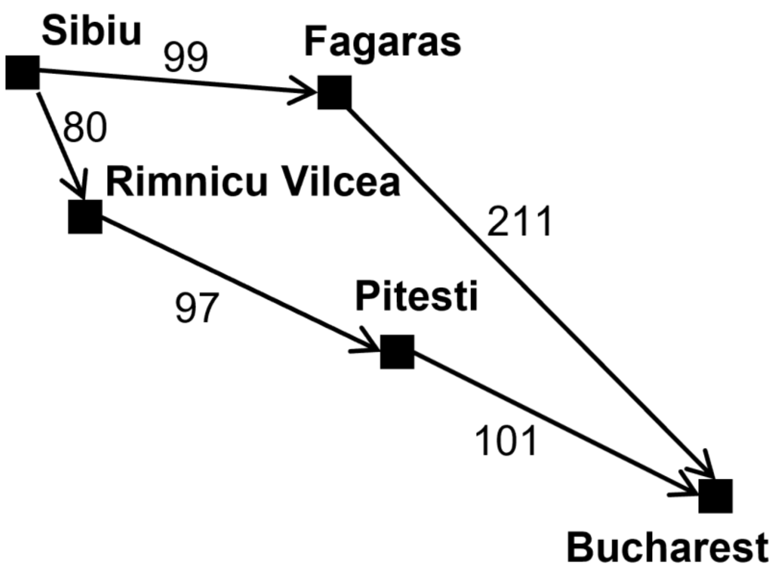
- Expands the node n with the *lowest path cost* $g(n)$
  - Done by storing the frontier as a priority queue ordered by g
  - *Path cost* $g(n)$: the cost of the path from the initial node to node n
- Example: get from Sibiu to Bucharest



Uniform-cost Search

Visited Nodes	Frontier
S	F, R
R	F, P
F	P, B_1
P	B_1, B_2
B_2	



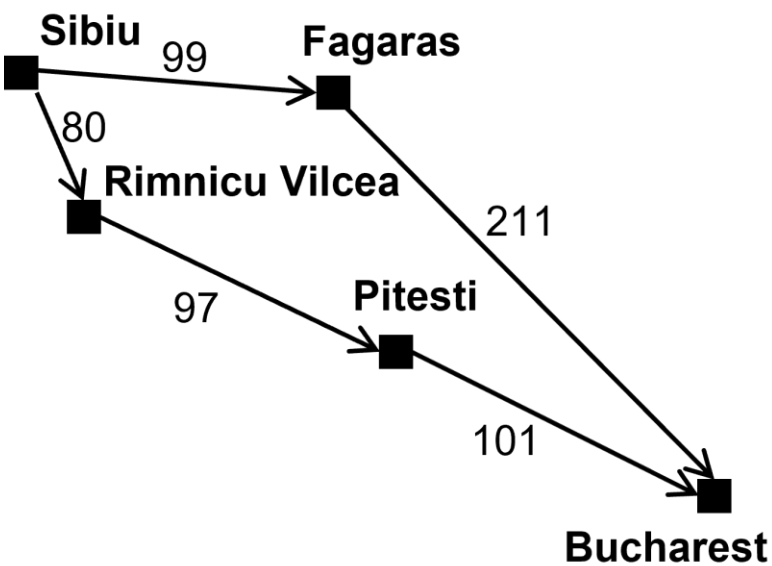


Solution path: $S \rightarrow R \rightarrow P \rightarrow B_2$

Solution path cost: 278

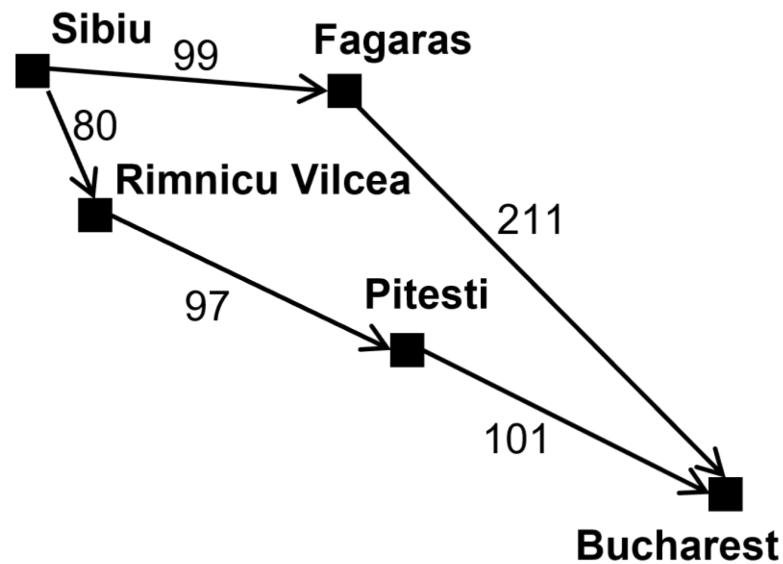
Uniform-cost Search

- Guided by path costs
- Does not care about the number of steps a path has



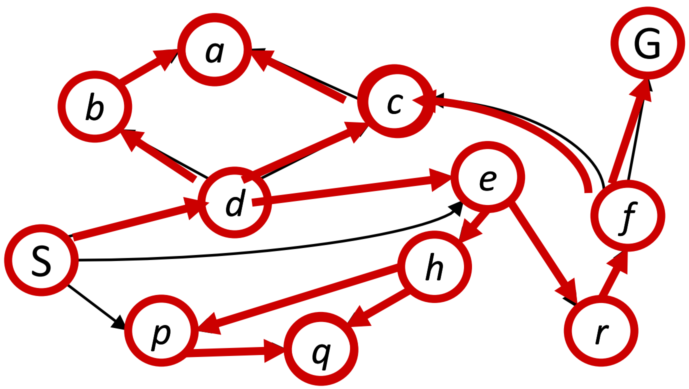
Uniform-cost Search

- Optimal or not?
  - Yes



- Completeness?
  - Guaranteed if the cost of every step is greater than a small positive value ε
  - If there's a path with an infinite sequence of zero-cost actions, then it will get stuck in an infinite loop

Depth-First Search



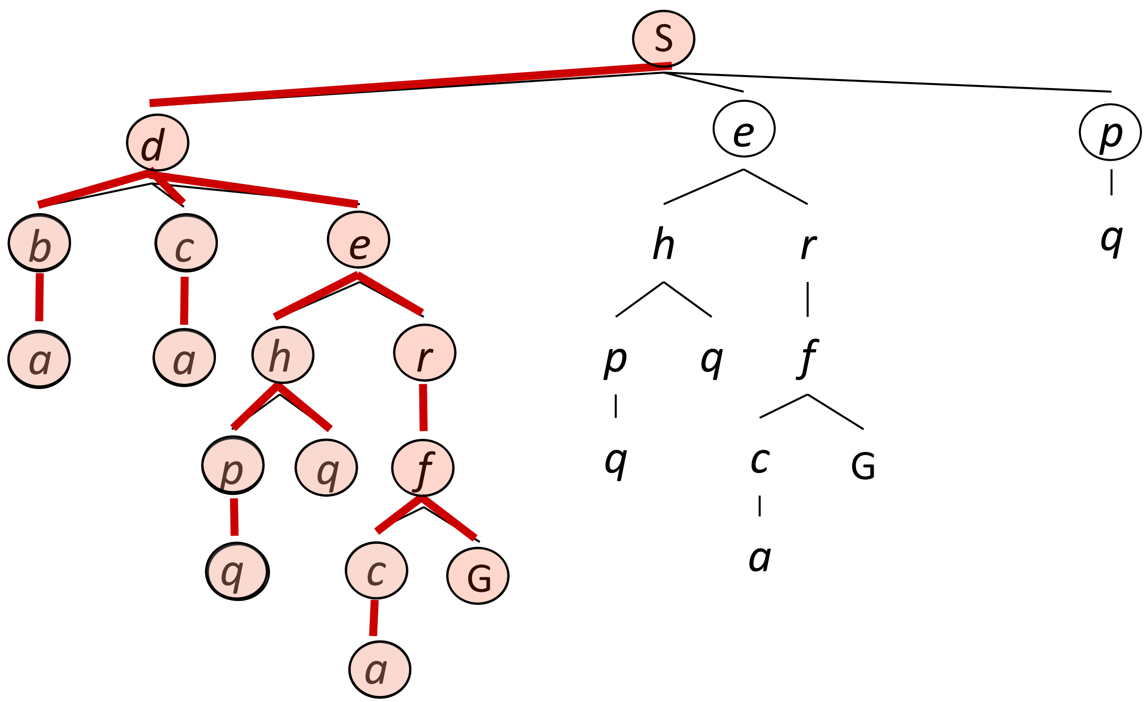
Initial State: S

Goal State: G

Explored: color circled

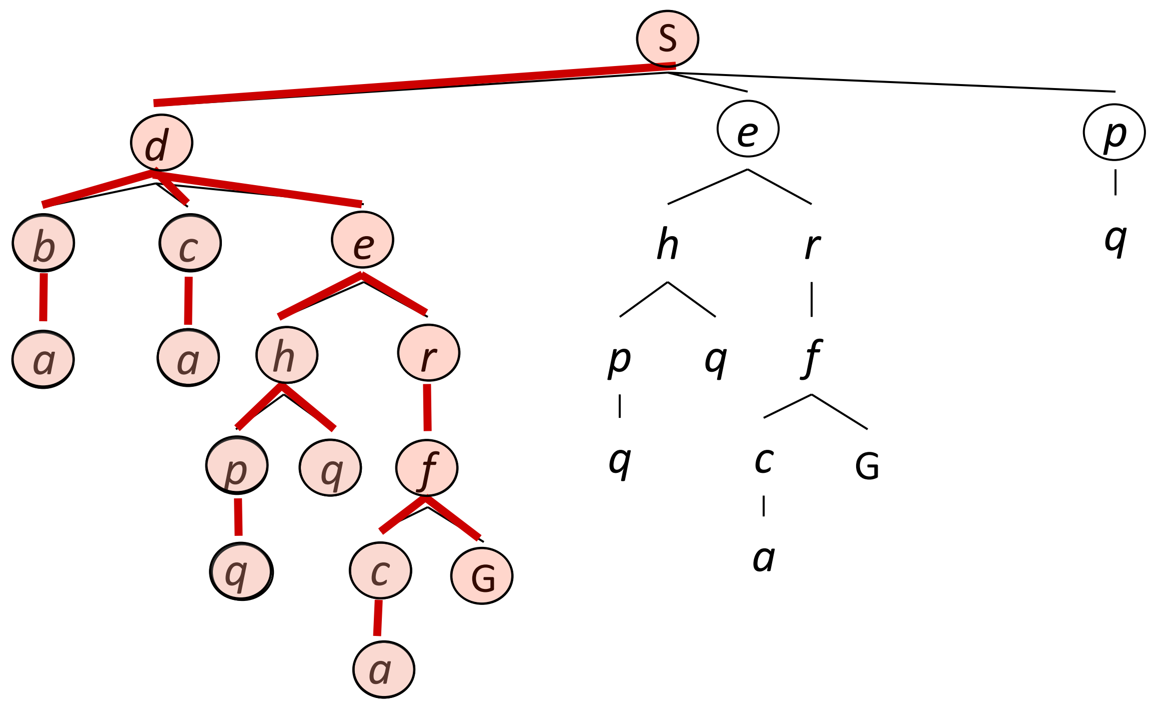
Frontier: white circled

Unexplored: uncircled



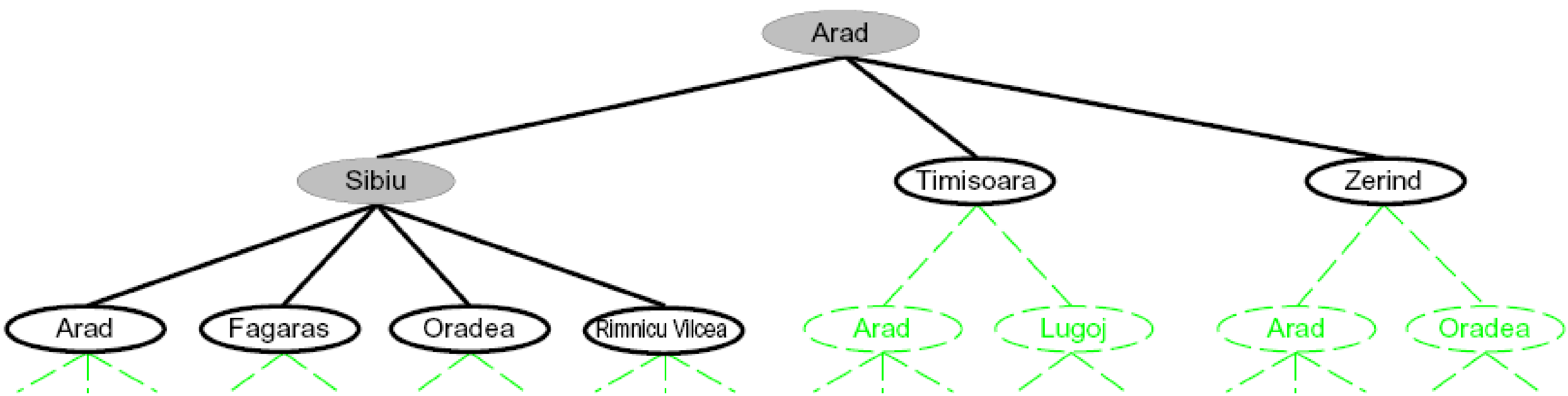
Depth-First Search

- Expands the deepest node in the current frontier
- LIFO stack – the most recently generated node is chosen for expansion
- Explored nodes with no descendants are removed from memory



Depth-First Search

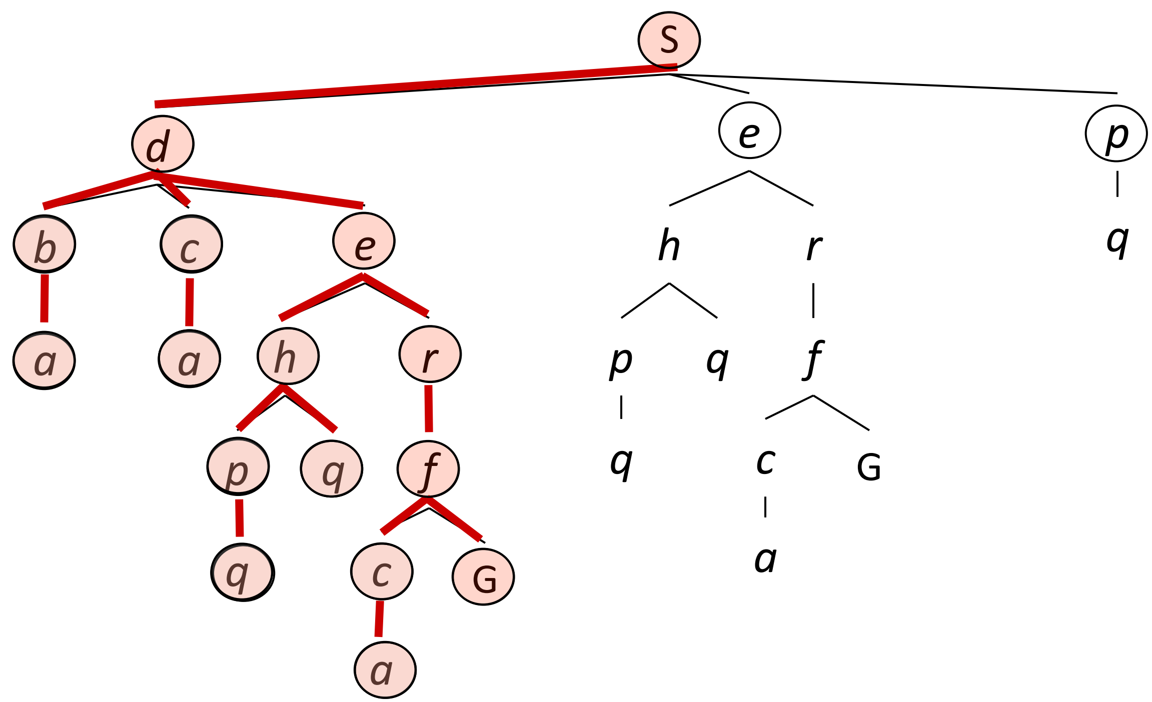
- Complete or not?
  - Depth-first Graph-search: Yes (avoids repeated states)
  - Depth-first Tree-search: No



- Arad-Sibiu-Arad-Sibiu loop forever!

Depth-First Search

- Optimal or not? (in terms of the cost of the solution path)



- Answer: No!
- The solution returned by the DFS will get to the goal state in 5 steps instead of 4 steps

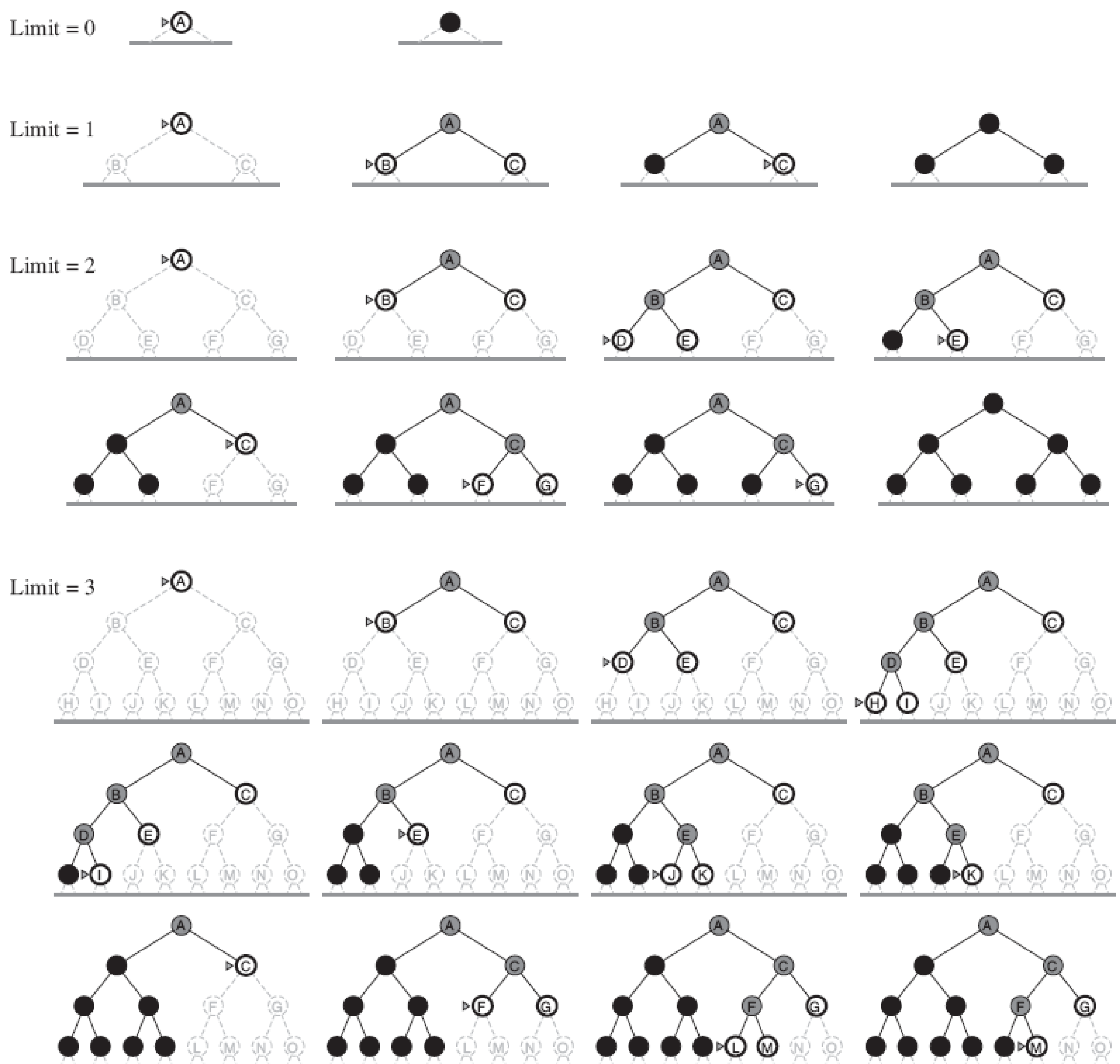
Depth-Limited Search

- Supply DFS with a predetermined depth limit l
  - Solves the infinite path problem
- Complete or not?
  - If $l < d$: incomplete! (e.g. when d is unknown)
- Optimal or not?
  - If $l > d$: Not guaranteed.

Iterative Deepening DFS

- Repeatedly applies depth-limited search with increasing limits l
- Terminates an iteration when a solution is found or if the depth-limited search returns *failure* (no solution for that depth limit)

Iterative Deepening DFS

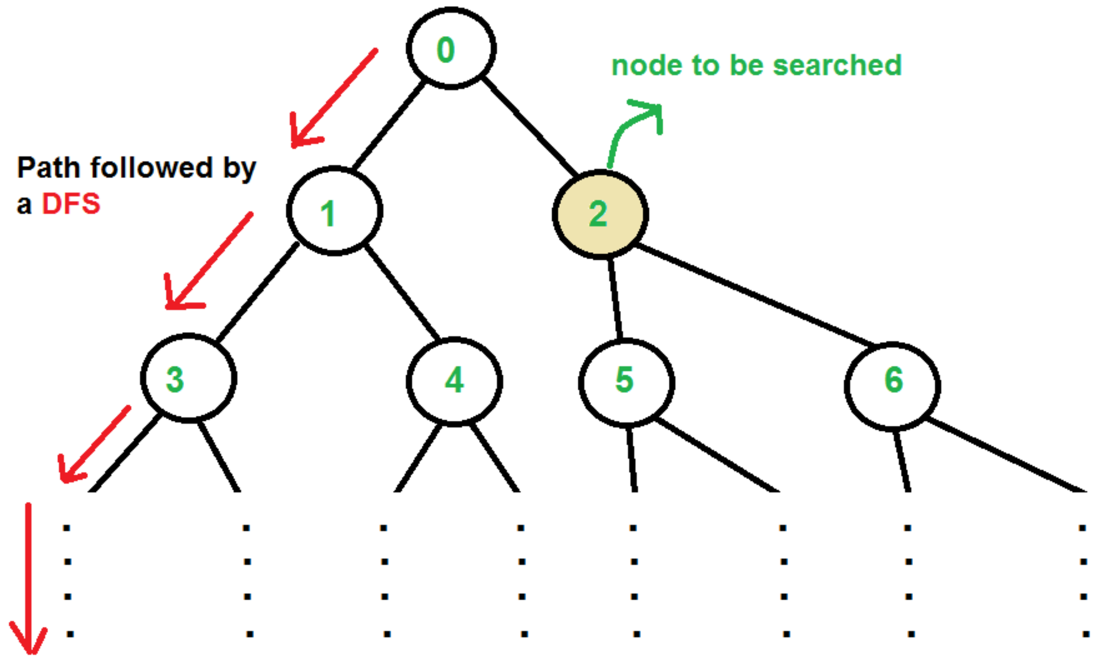


Four iterations of iterative deepening DFS on a binary tree

Suppose: M is the goal node

Black circles: nodes removed from memory

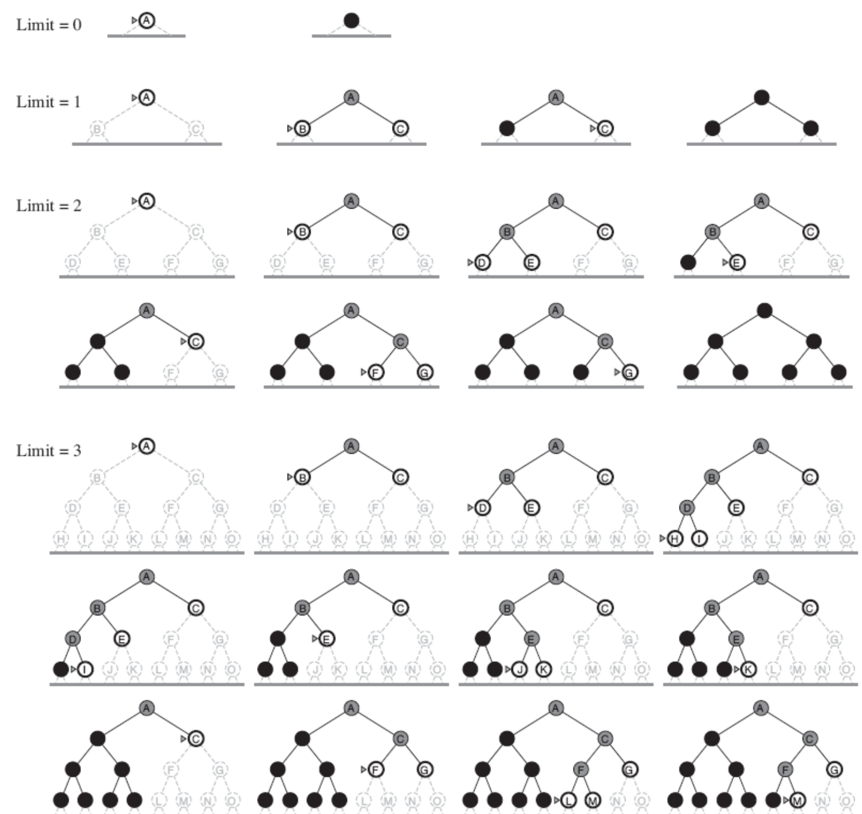
Iterative Deepening DFS



- We want to find node '2' of the given "deep" tree.
- A DFS starting from node '0' will dive left, towards node 1 and so on
- Hence, a DFS wastes a lot of time in coming back to node 2
- An Iterative Deepening DFS overcomes this and quickly finds the desired node.

Iterative Deepening DFS

- Complete? Yes when b is finite
- Optimal? Yes in the sense that it can always find the shallowest solution.



Iterative Deepening DFS

- Combines the benefits of DFS and BFS
  - Benefit of BFS: optimal (shallowest solution)
  - Benefit of DFS: space complexity

Informed Search Strategies

- The search strategies have extra information regarding how “close” a node is to a goal node

- Can find solutions more efficiently than uninformed search strategies

Heuristic Function $h(n)$

- Heuristic function

  $h(n)$= estimated cost of the cheapest path from node n to a goal state

- Let $h(n)$ be
  - Nonnegative
  - Problem-specific functions
  - Define: if n is a goal node, then $h(n) = 0$

Greedy Search

- Expands the node that seems closest to the goal
- Evaluates nodes by using a heuristic function $h(n)$
- Example: route-finding problem in Romania
  - Use straight-line distance heuristic h_{SLD}

The straight-line distance to Bucharest

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Greedy Search

The straight-line distance to Bucharest

Arad	366	Mehadia	241
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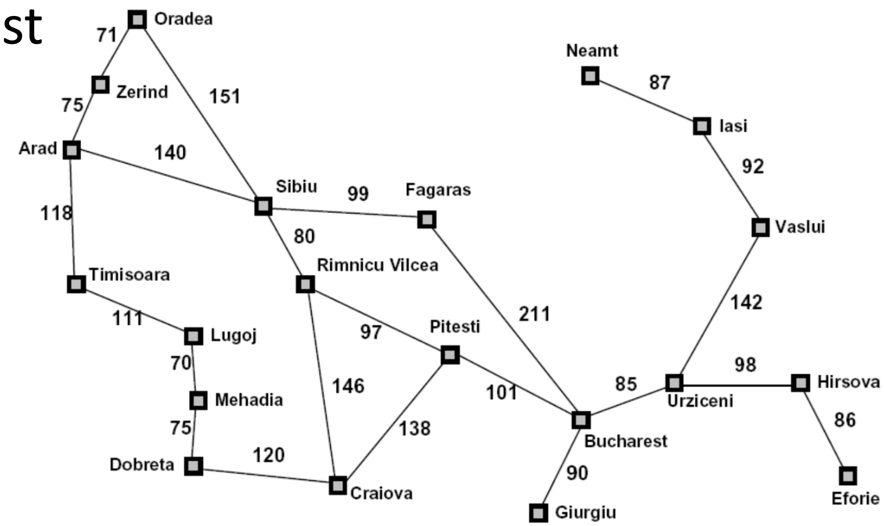
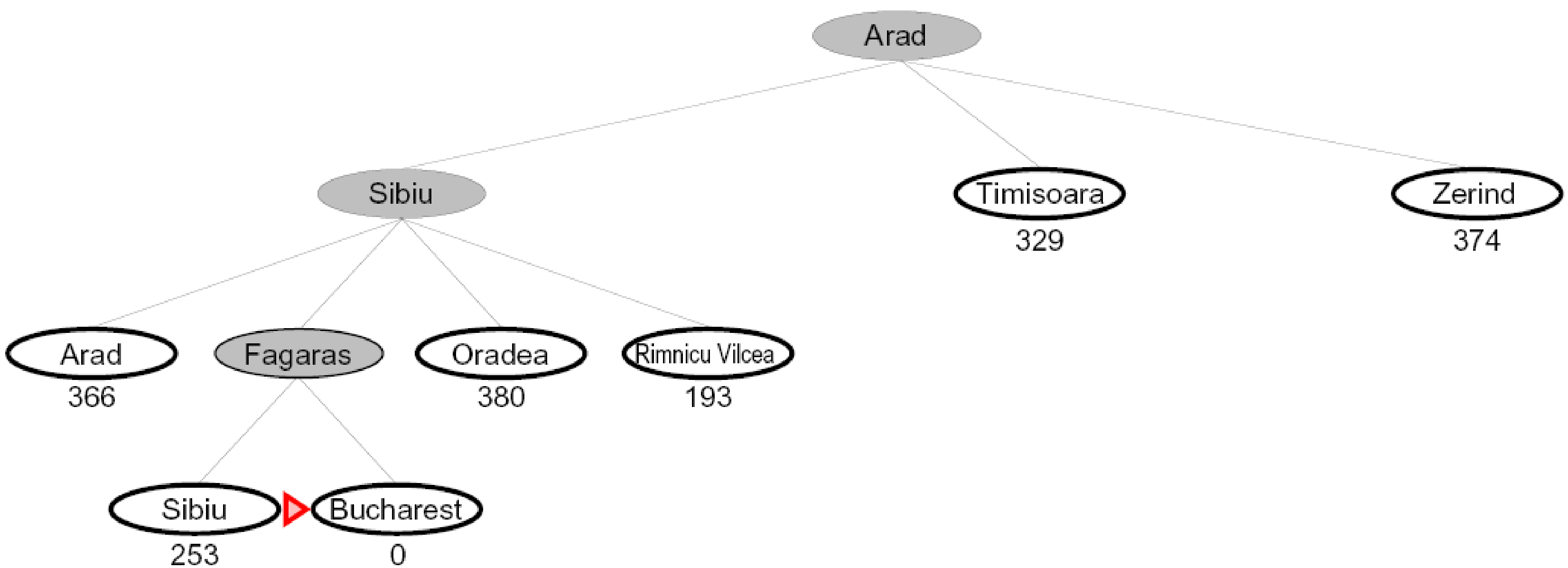




Greedy Search

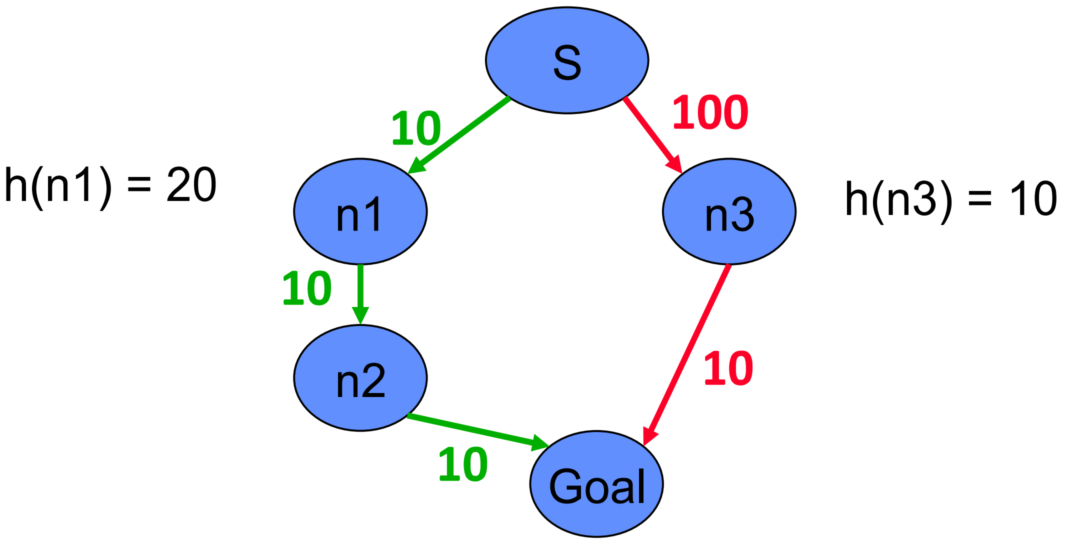
- Why is it called “greedy”?
  - At each step, it tries to get as close to the goal as it can

Greedy Search

- Not optimal
  - It ignores the cost of getting to n
  - Can be led astray exploring nodes that cost a lot but seem to be close to the goal

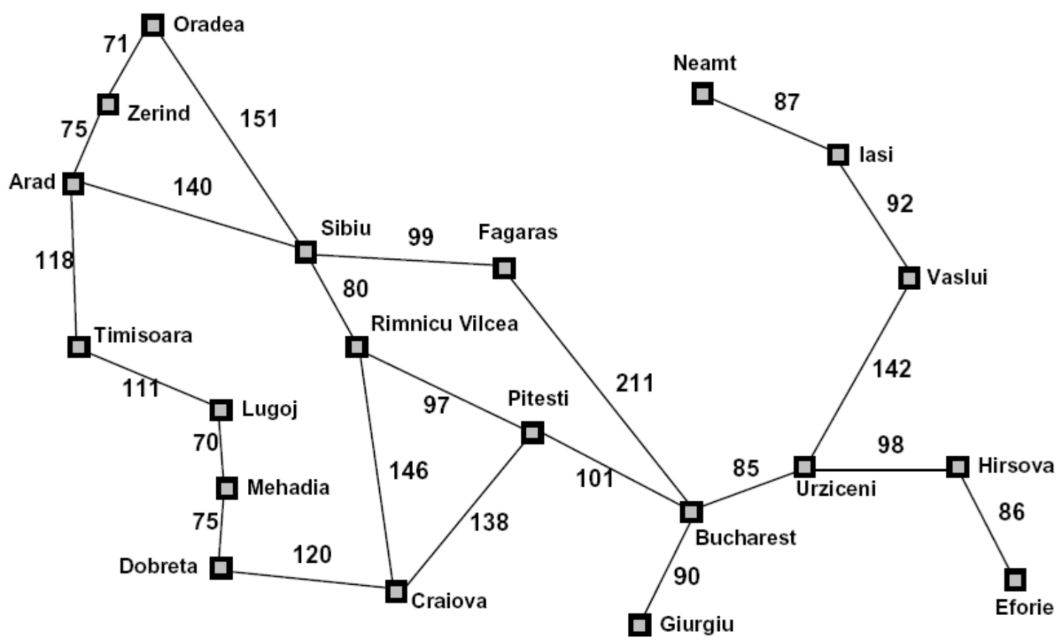
S to n1: → step cost = 10

S to n3: → step cost = 100



Greedy Search

- Completeness
  - Greedy tree search: Incomplete even in a finite state space (does not guarantee to find a solution)



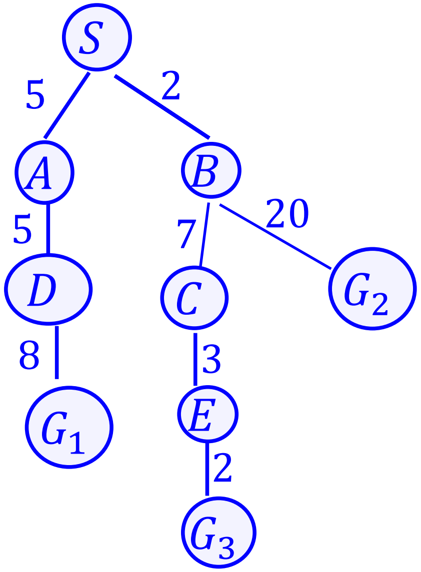
  - The graph search version: Complete in finite state spaces, but not in infinite state spaces

A* Search

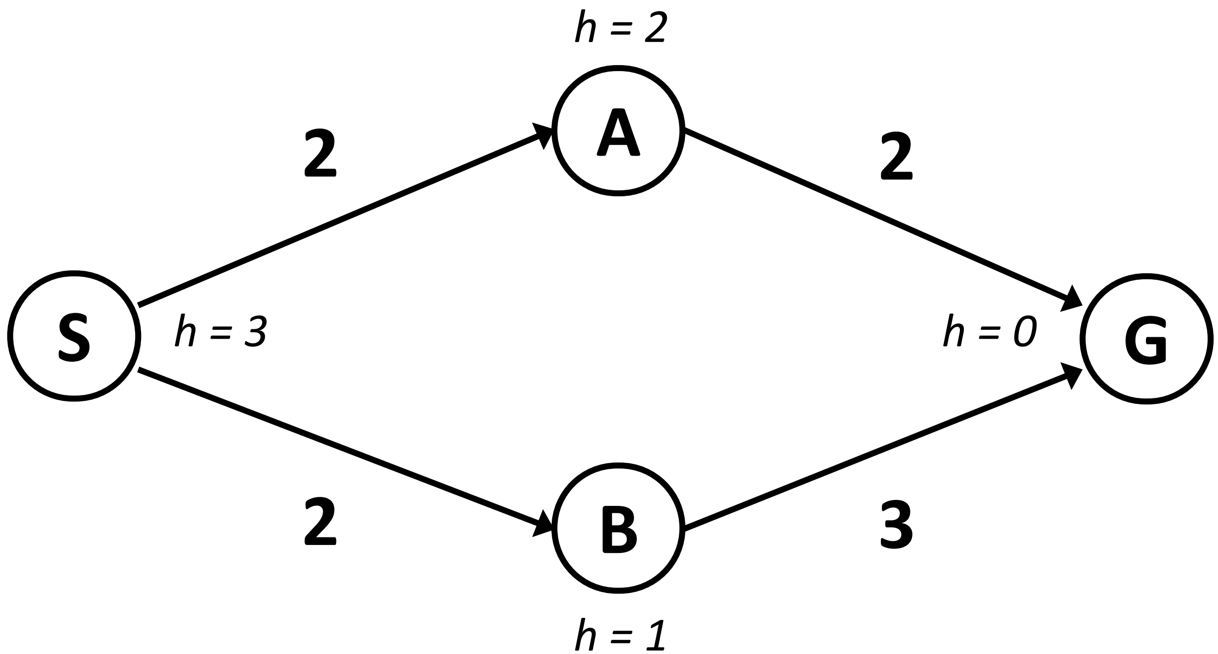
- Minimizes the estimated total cost of a solution path

$$f(n) = g(n) + h(n)$$

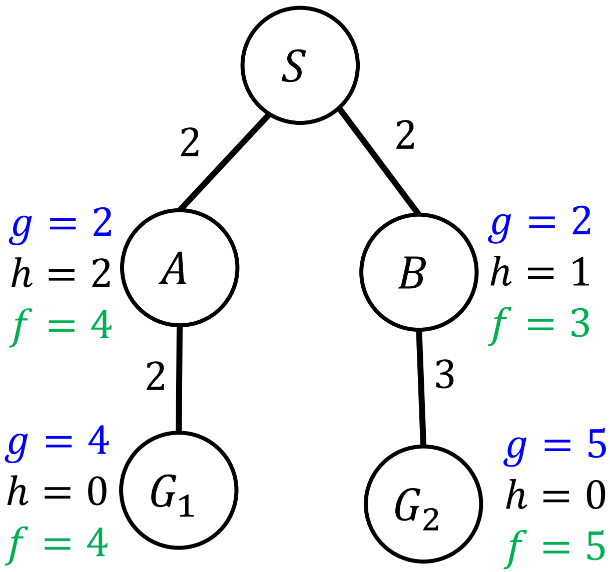
  - $g(n)$ - the path cost from the initial node to node n
  - $h(n)$ - the estimated cheapest cost to get from node n to the goal node
  - $f(n)$ – estimated total cost of the cheapest path that continuous from node n to a goal



A^* Search Example



- Start from S, G is the goal
- Expanded nodes in order: S, B, A, G
- Solution path: S->A->G
- Solution path cost: 4

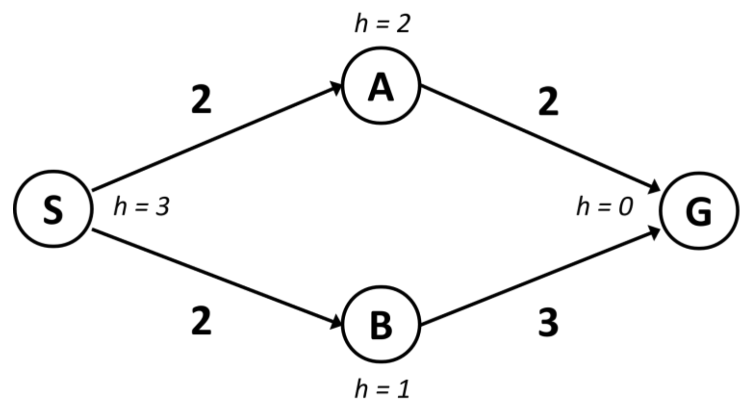


$$A^*: f(n) = g(n) + h(n)$$

Visited	Frontier
S	A, B
B	A, G_2
A	G_2, G_1
G_1	

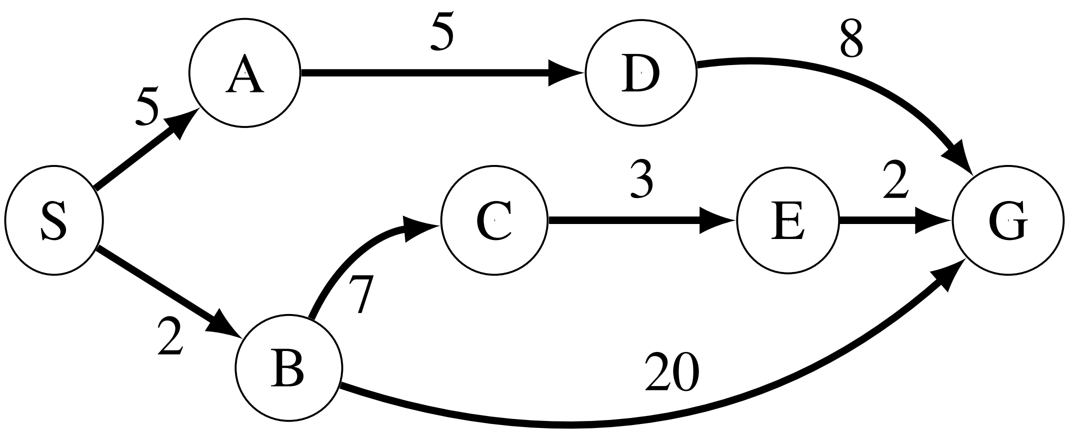
Solution path: $S \rightarrow A \rightarrow G_1$

Solution path cost: 4

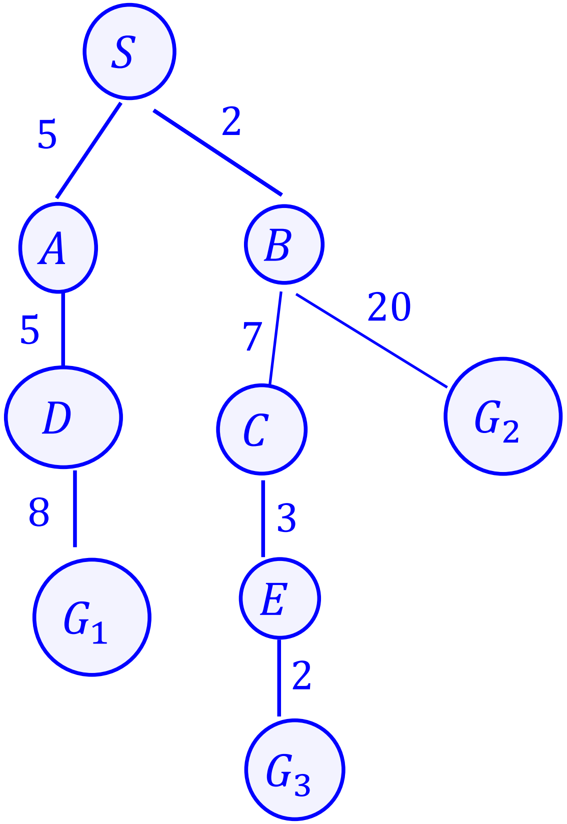


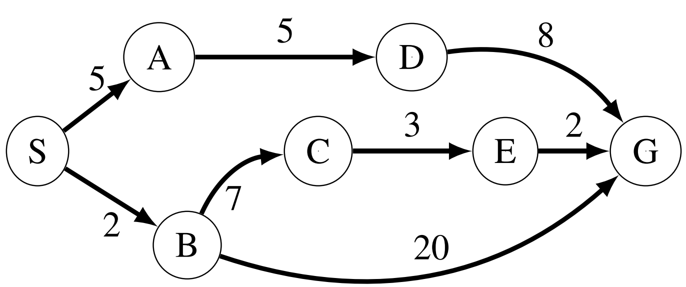
BFS Example

- S is the start node, G is the goal node, use BFS to find a path from S to G. List the expanded nodes in order, give the solution path, and solution path cost. Use alphabetical order to break ties.



- Answer:
- Expanded nodes: S,A,B,D,C,G
- Solution path: SBG
- Path cost: 22





BFS

Visited:

S, A, B, D, C, G_2

Solution path:

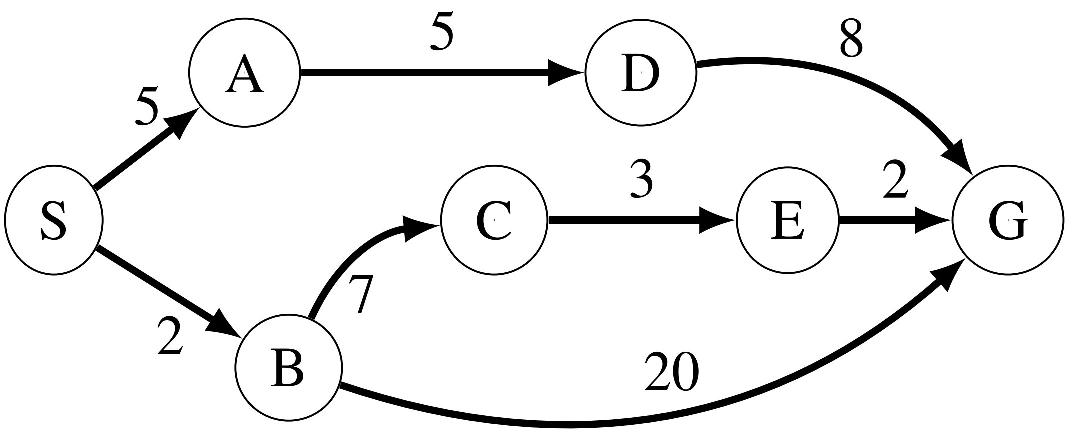
$S \rightarrow B \rightarrow G_2$

Sol. path cost:

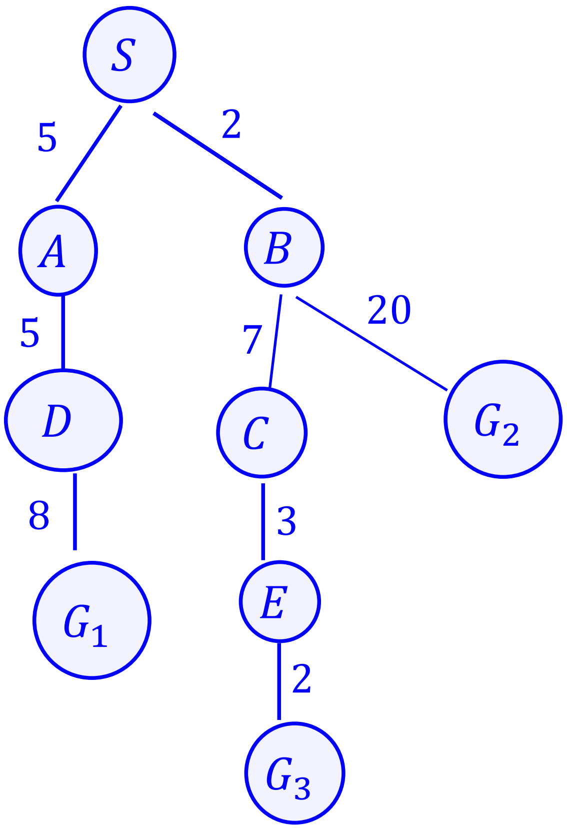
$2 + 20 = 22$

DFS Example

- Solve the same problem by depth-first search.



- Answer:
- Expanded nodes: S,A,D,G
- Solution path: S,A,D,G
- Path cost: 18



DFS

visited:

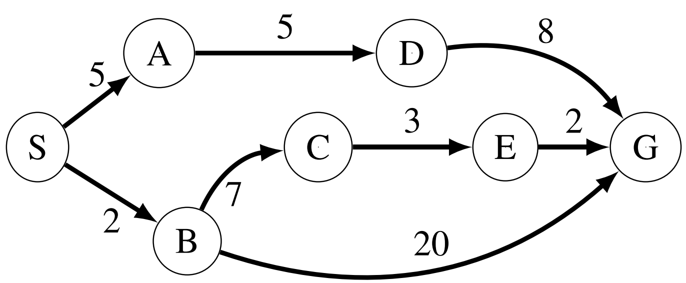
$S \to A \to D \to G_1$

sol. path:

$S \to A \to D \to G_1$

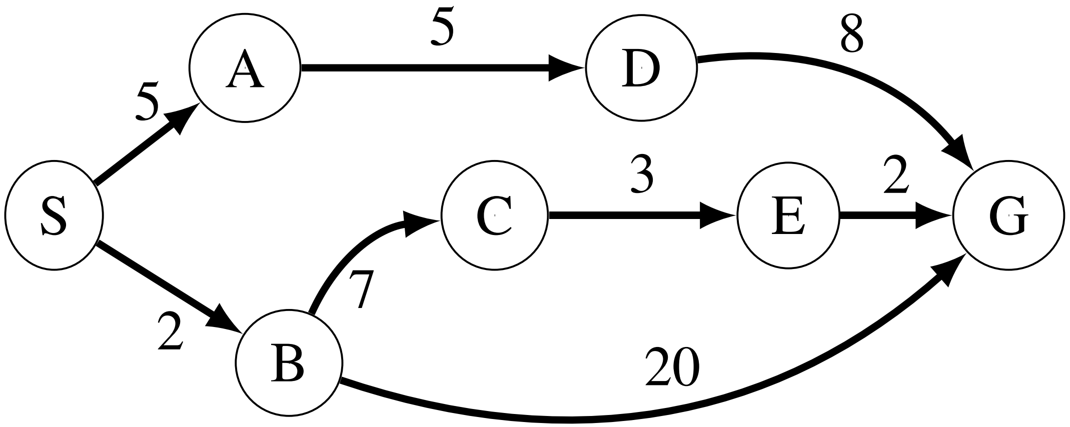
sol. path cost:

$5 + 5 + 8 = 18$

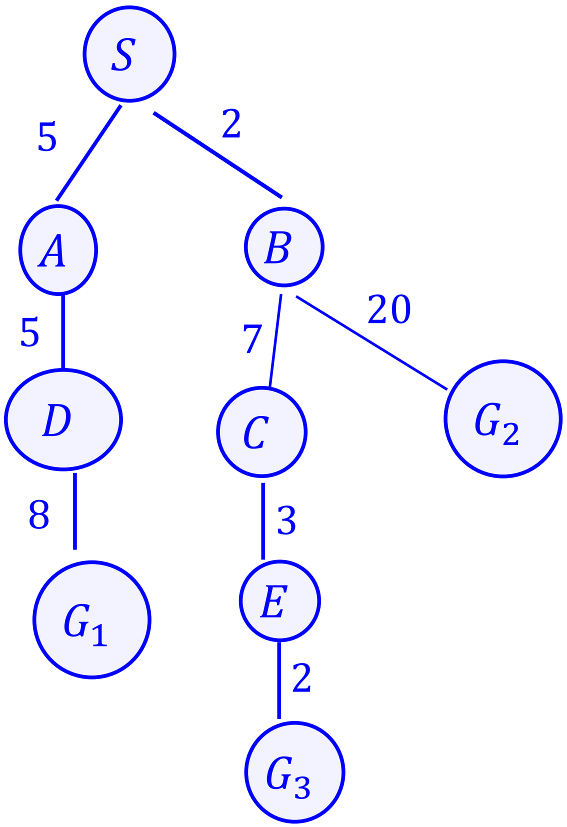


IDDFS Example

- Solve the same problem by the Iterative Deepening DFS strategy.



- Answer:
- Expanded nodes: S; SAB; SADBCG.
- Solution path: SBG
- Path cost: 22



IDDFS

Visited:

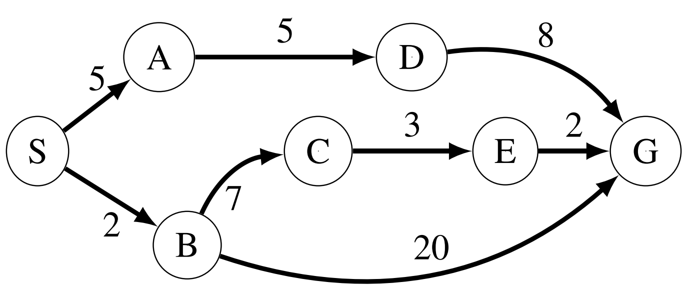
$S;$

$S, A, B;$

S, A, D, B, C, G_2

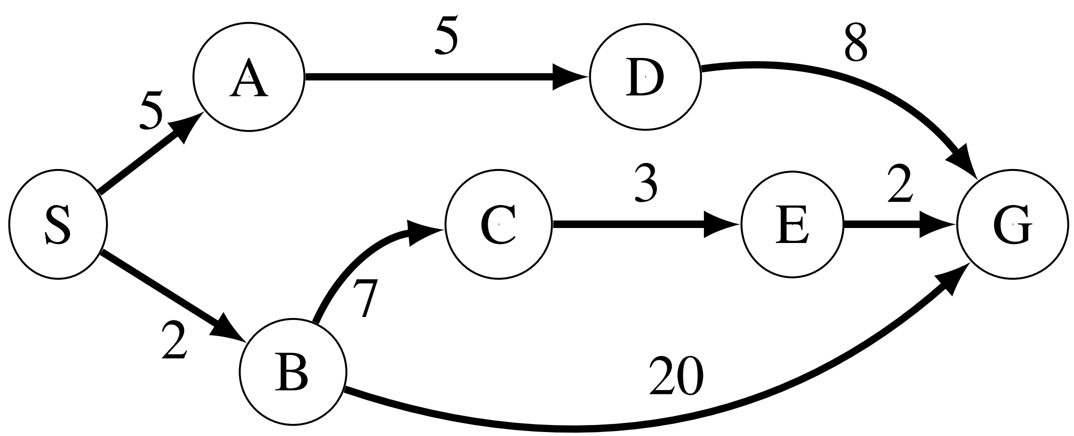
sol. path: $S \rightarrow B \rightarrow G_2$

sol path cost: $2 + 20 = 22$

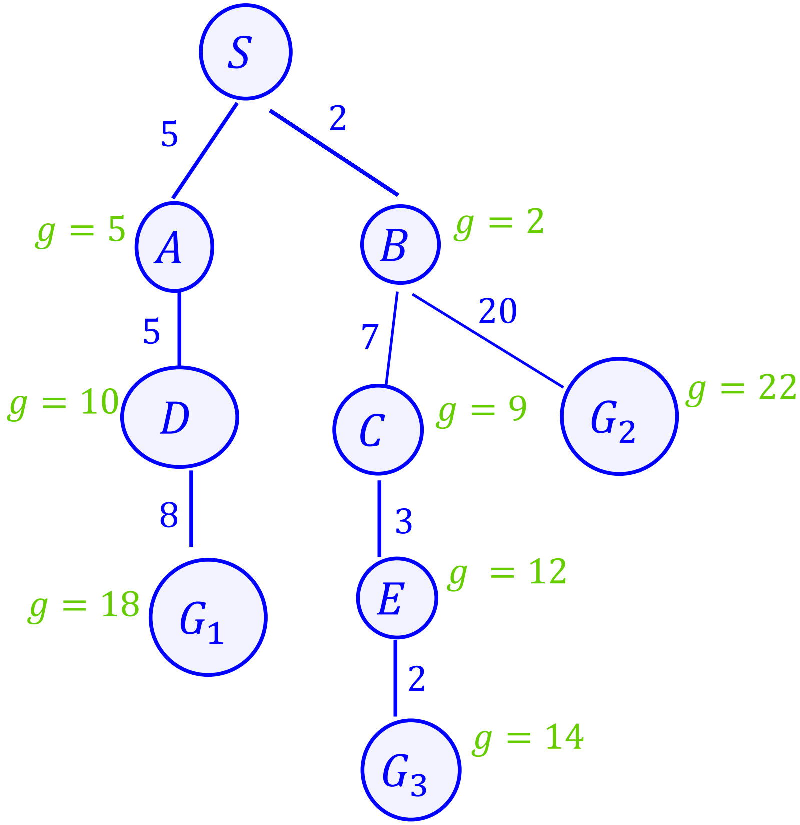


UCS Example

- Solve the same problem by the uniform-cost search.



- Answer:
- Expanded nodes: S,B,A,C,D,E,G
- Solution path: SBCEG
- Path cost: 14

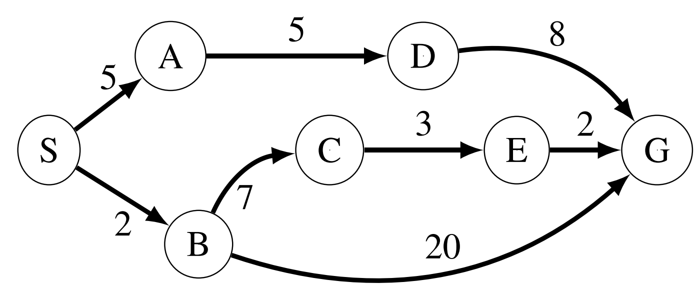


UCS: g valves.

Visited	Frontier
S	A, B
B	A, C, G_2
A	C, G_2, D
C	G_2, D, E
D	G_2, E, G_1
E	G_2, G_1, G_3
G_3	

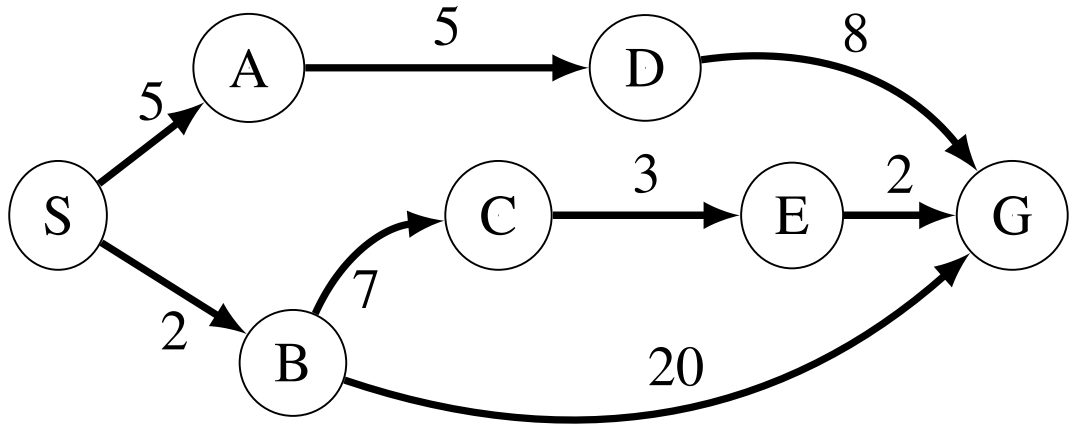
Sol. path: $S \rightarrow B \rightarrow C \rightarrow E \rightarrow G_3$

Sol. path cost: 14



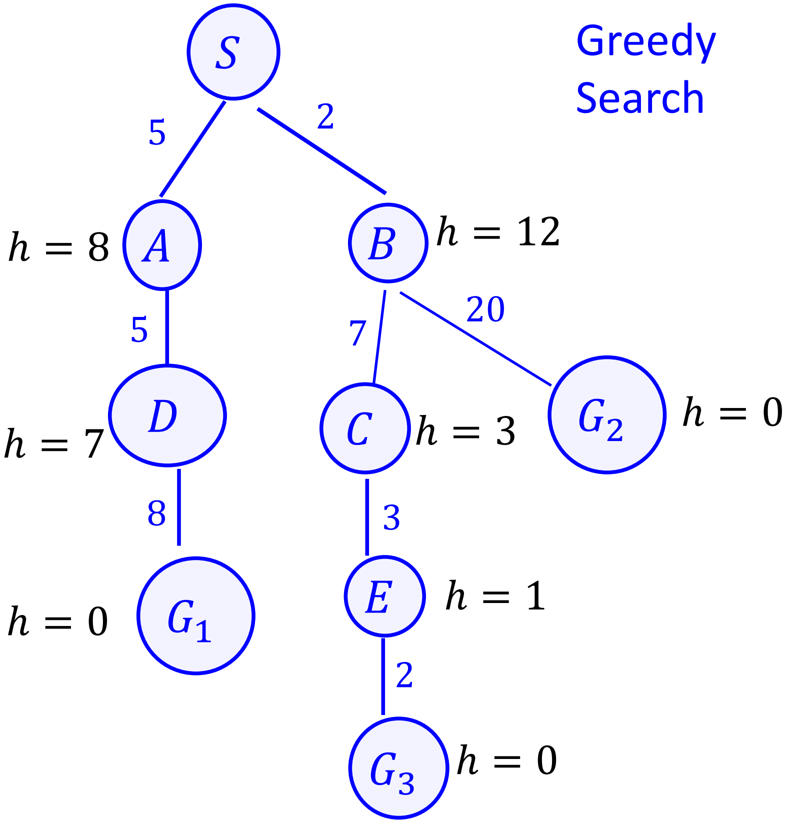
Greedy Search Example

- Solve the same problem by greedy search. The heuristic function values are given.



- Answer:
- Expanded nodes: S,A,D,G
- Solution path: S,A,D,G
- Path cost: 18

h(S)=12, h(A)=8,
h(B)=12, h(C)=3,
h(D)=7, h(E)=1,
h(G)=0.

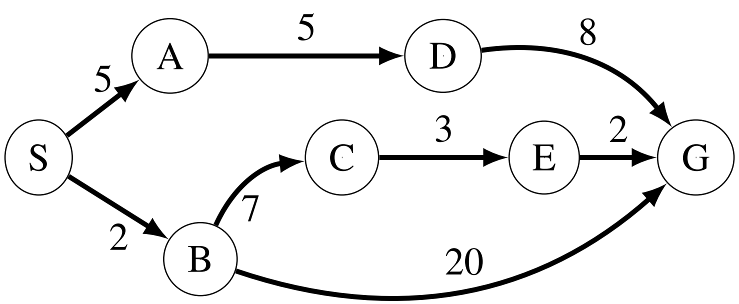


Greedy Search

Visited	Frontier
S	A, B
A	B, D
D	B, G_1
G_1	

solution path: $S \rightarrow A \rightarrow \mathrm{D} \rightarrow G_1$

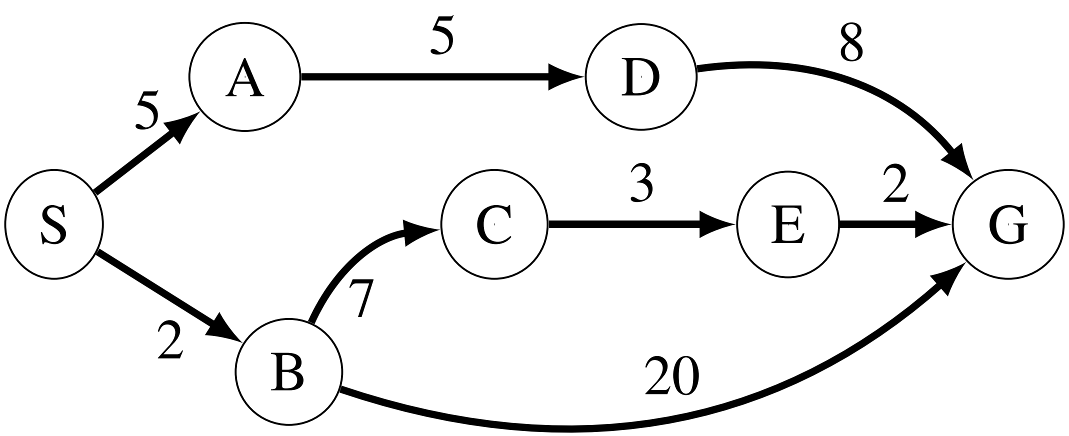
solution path cost: 18



h(S)=12, h(A)=8,
h(B)=12, h(C)=3,
h(D)=7, h(E)=1,
h(G)=0.

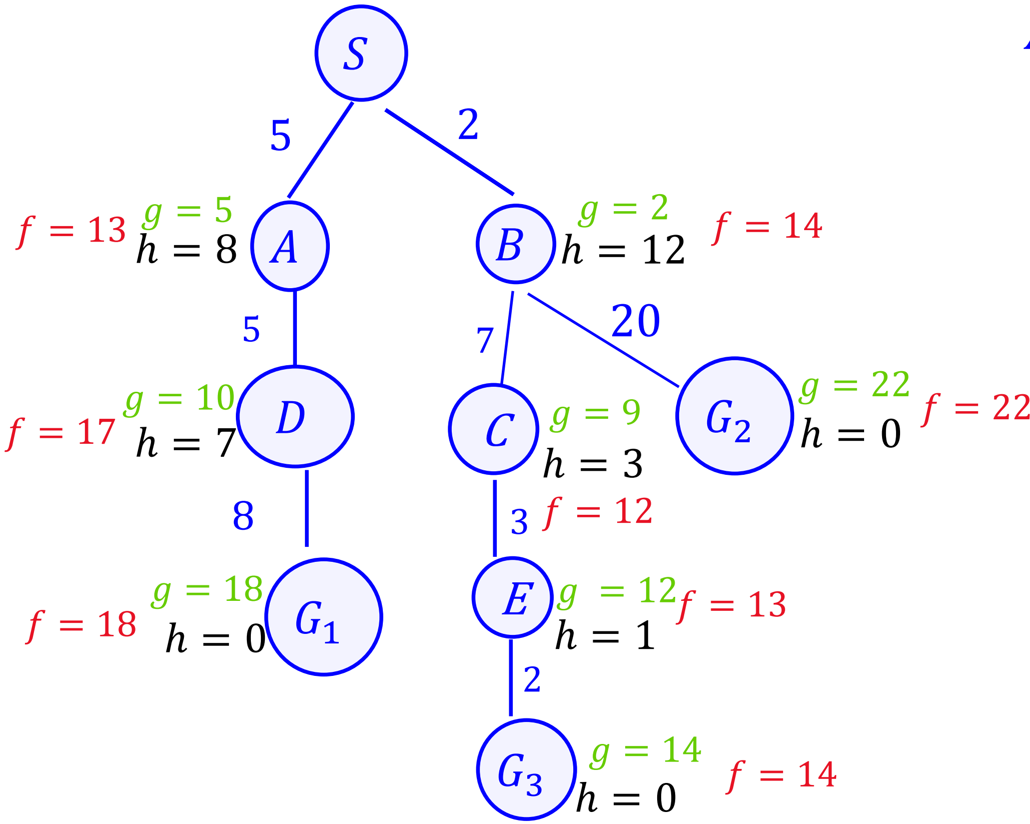
A* Search Example

- Solve the same problem by the A* search strategy.



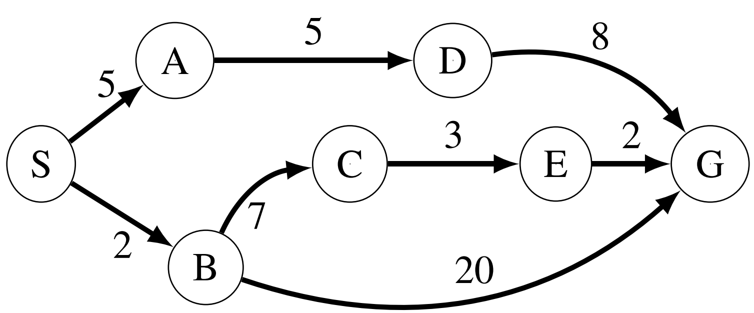
- Answer:
- Expanded nodes: S,A,B,C,E,G
- Solution path: SBCEG
- Path cost: 14

h(S)=12, h(A)=8,
h(B)=12, h(C)=3,
h(D)=7, h(E)=1,
h(G)=0.



$A^*: f(n) = g(n) + h(n)$

Visited	Frontier
S	A, B
A	B, D
B	D, C, G_2
C	D, G_2, E
E	D, G_2, G_3
G_3	



Sol. path: $S \rightarrow B \rightarrow C \rightarrow E \rightarrow G_3$

Sol. path cost: 14

h(S)=12, h(A)=8,
h(B)=12, h(C)=3,
h(D)=7, h(E)=1,
h(G)=0.