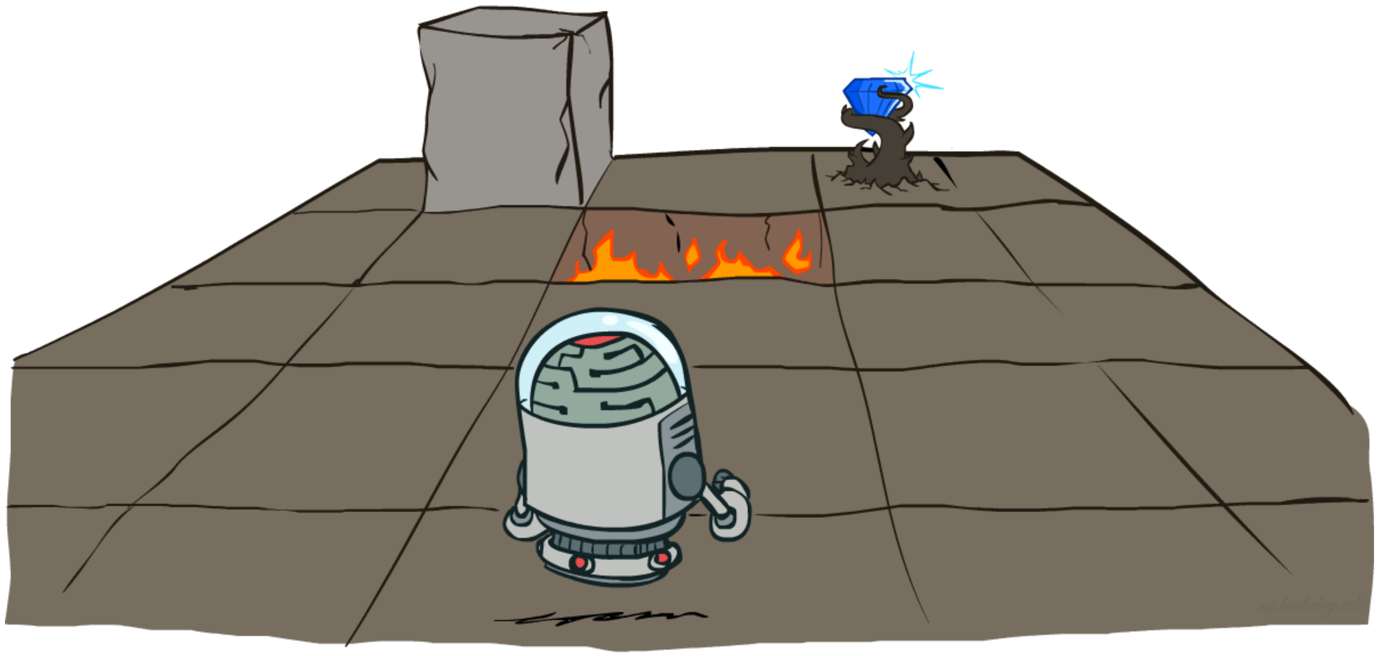
Markov Decision Processes



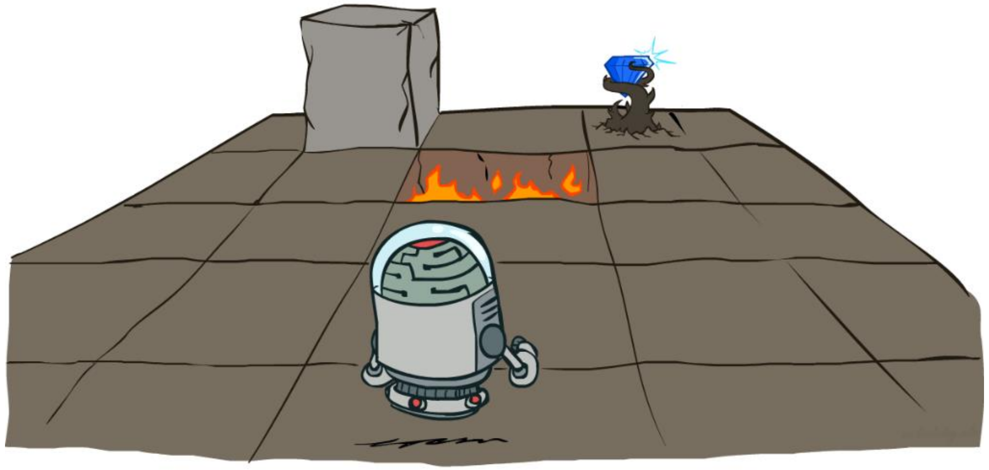
These slides were adapted from the MDP lecture slides of cs188 developed at UC Berkeley, http://ai.berkeley.edu.

Santa Clara University

Stochastic Environment

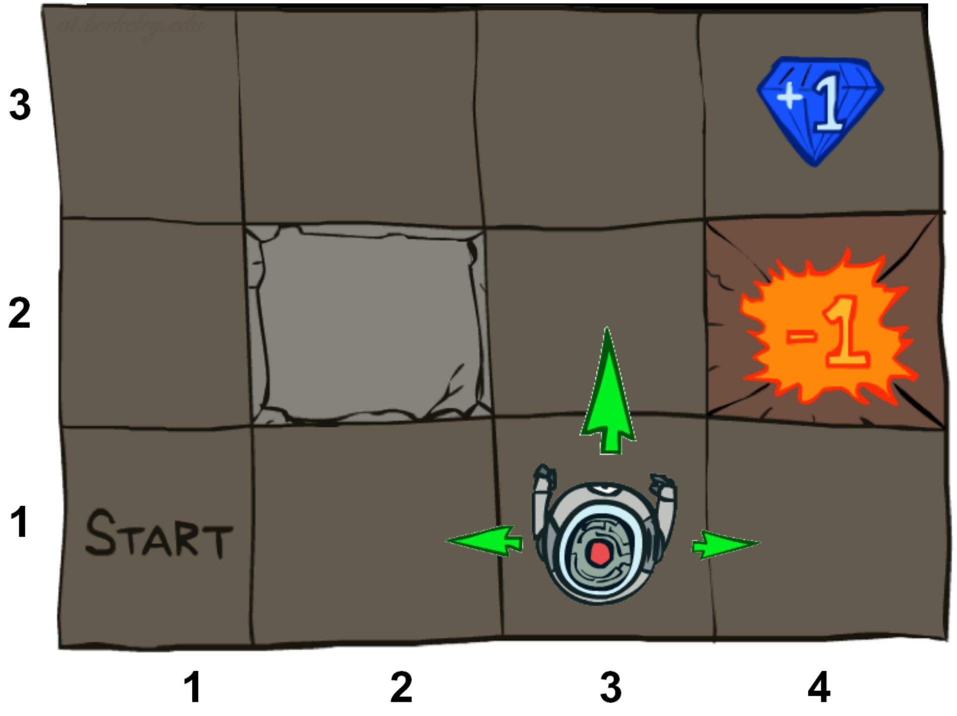
Motivation of Markov Decision Processes

- Another way to solve the decision-making problem for stochastic games
- Sometimes it's difficult to use the Minimax algorithm to determine actions, for example
  - When the game tree is endless
  - When the time spent to finish the game (finish the task) also affects the payoff



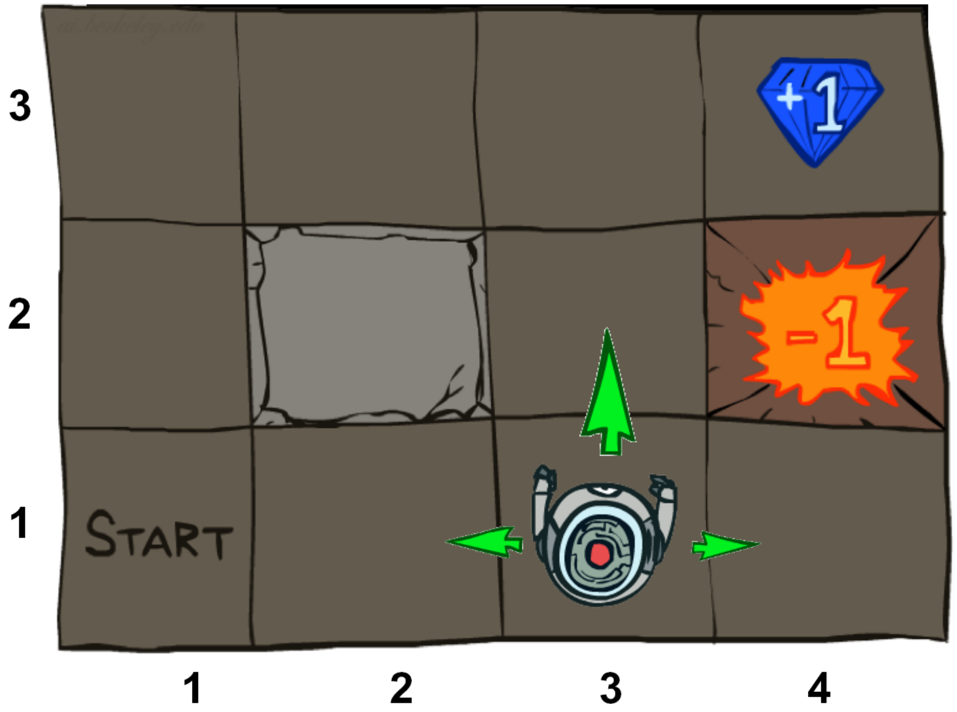
Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North
    (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put



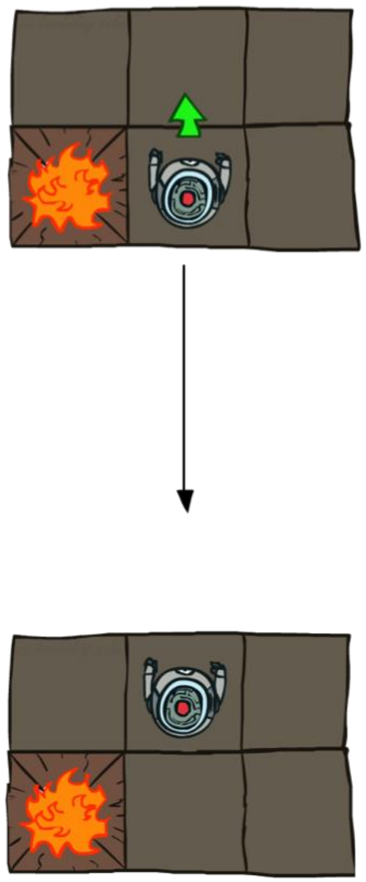
Example: Grid World

- The agent receives rewards each time step
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards by the end of the game

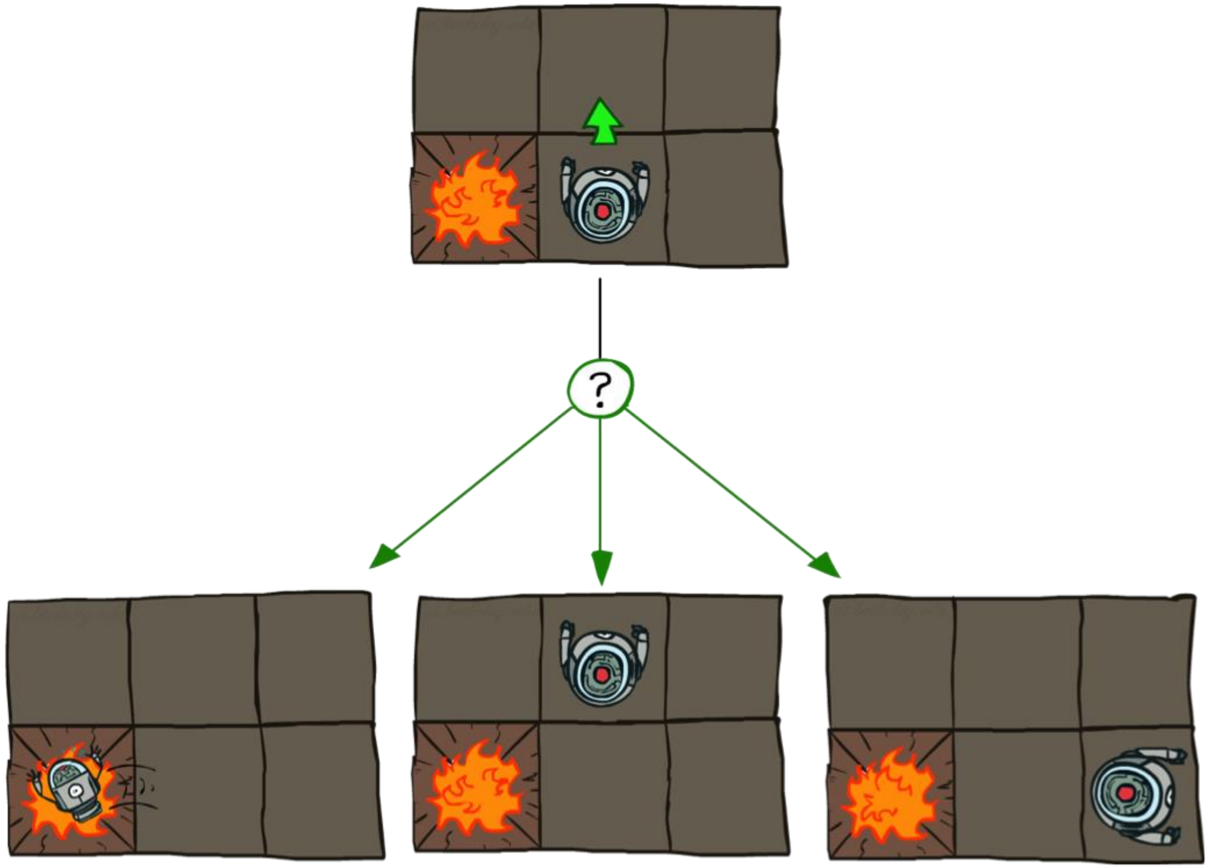


Grid World Actions

Deterministic Grid World



Stochastic Grid World



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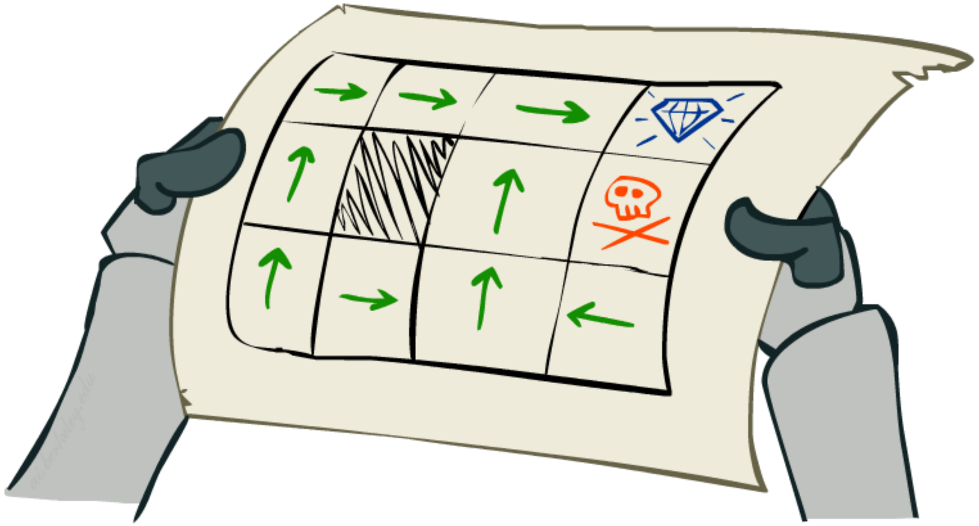
The Objective

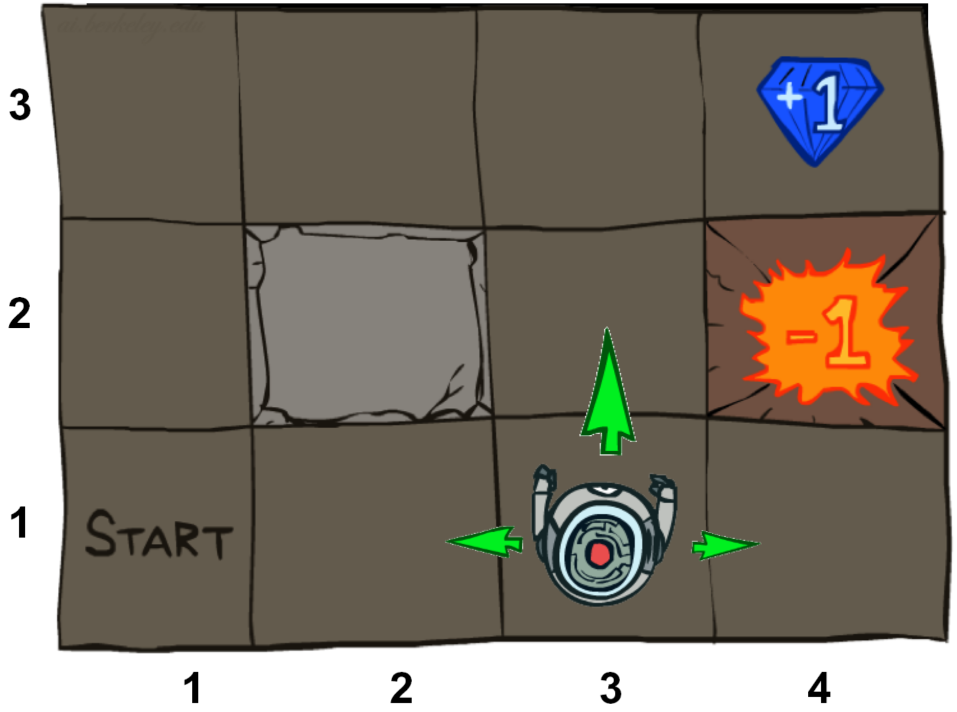
- In deterministic single-agent search problems, we wanted an optimal plan, or a sequence of actions, that takes the agent from the start to a goal state

- For a Markov Decision Process, we want to determine **the best action** for each state in the state space



Markov Decision Processes

- **An MDP is defined by:**
  - A set of states s ∈ S
  - A set of actions a ∈ A
  - A transition function T(s, a, s')
    - Probability that taking action a from state s leads to state s', i.e., P(s'| s, a)
    - Also called the model or the dynamics



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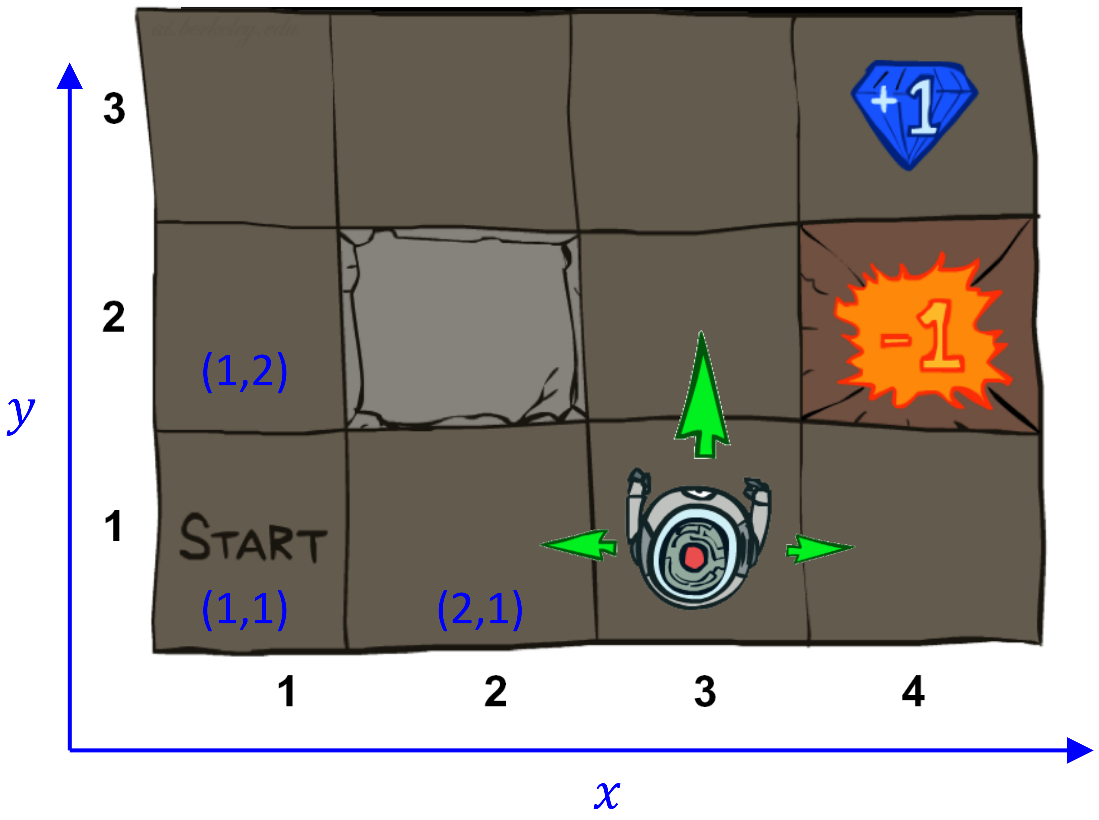
Markov Decision Processes

- **Transition function T(s, a, s')**
  - For example
  - T(s=(1,1), a=North,s'=(1,2))=?
  - T(s=(1,1), a=North,s'=(2,1))=?
  - T(s=(1,1), a=North,s'=(1,1))=?

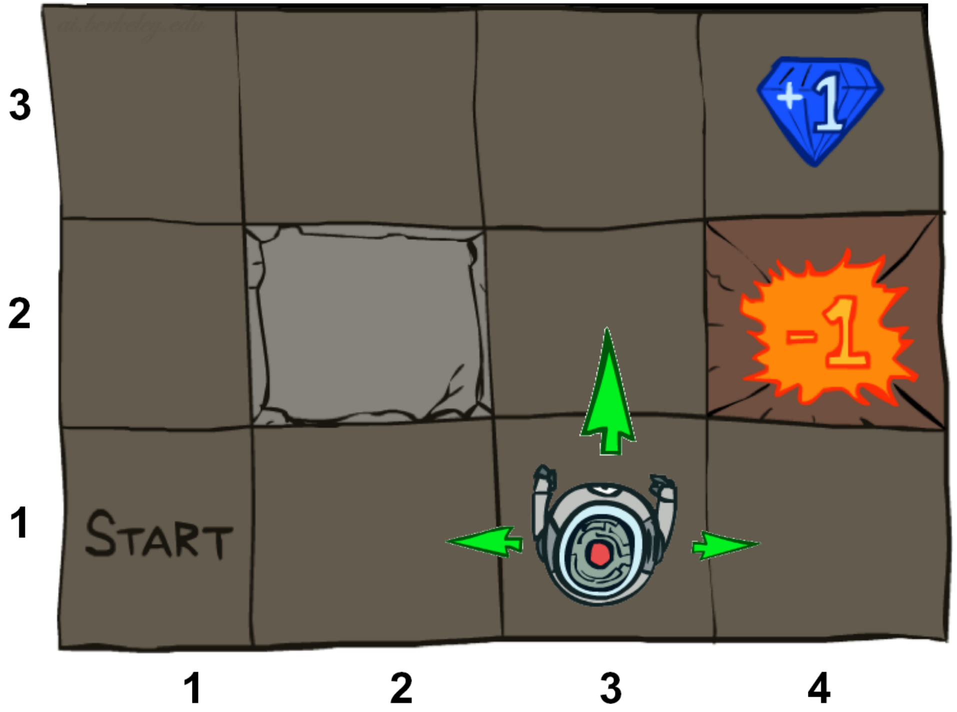
  - T(s=(1,1), a=East,s'=(2,1))=?
  - T(s=(1,1), a=East,s'=(1,2))=?
  - T(s=(1,1), a=East,s'=(1,1))=?

  - That is, for any possible combination (s,a,s'), we need to define T(s,a,s')



Markov Decision Processes

- Example: when the agent takes an action
  - 80% of time he lands in the desired square;
  - 20% of time he lands in the square in an orthogonal direction of the desired direction.



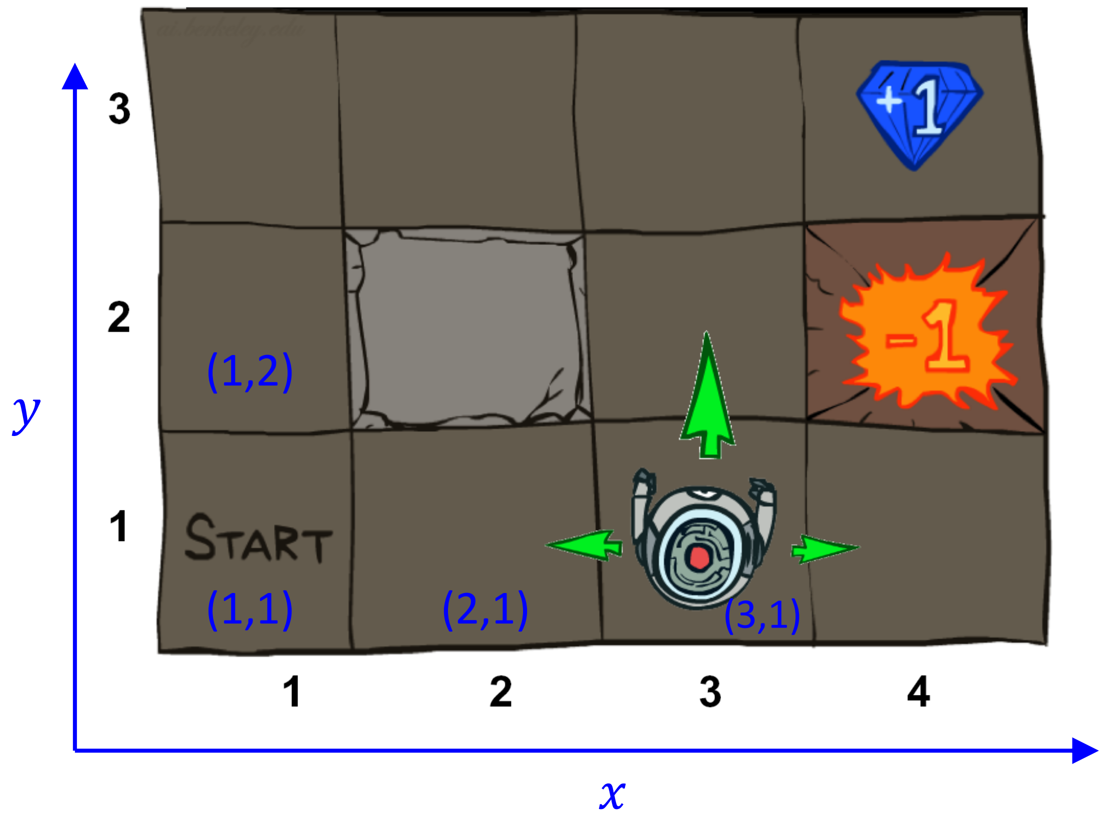
Markov Decision Processes

- What are the following transition probabilities? Let the location be (x,y)

  - T(s,North,s'), when s= (1,1), s'=(2,1)
  - Answer: 0.1

  - T(s,East,s'), when s=(1,1), s'=(2,1)
  - Answer: 0.8

  - T(s,East,s'), when s=(2,1), s'=(3,1)
  - Answer: 0.8
  - Where else can the agent land?
  - Answer: stay at (2,1)

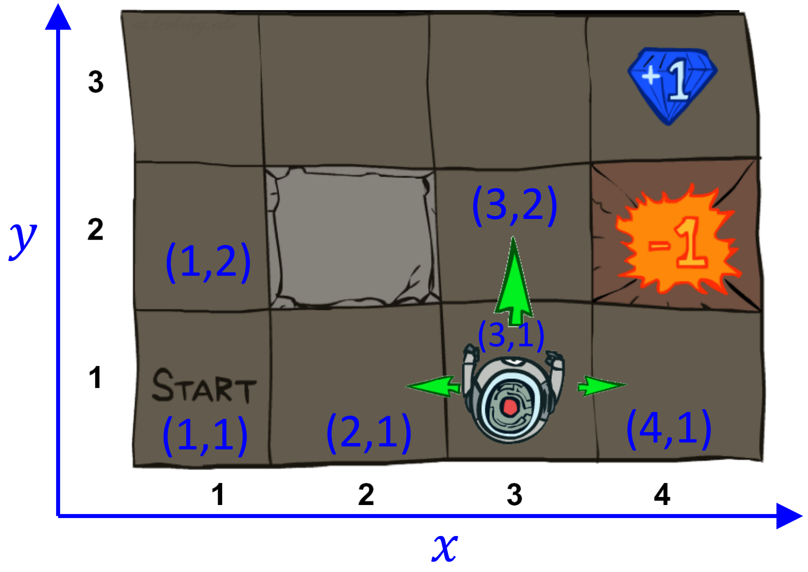


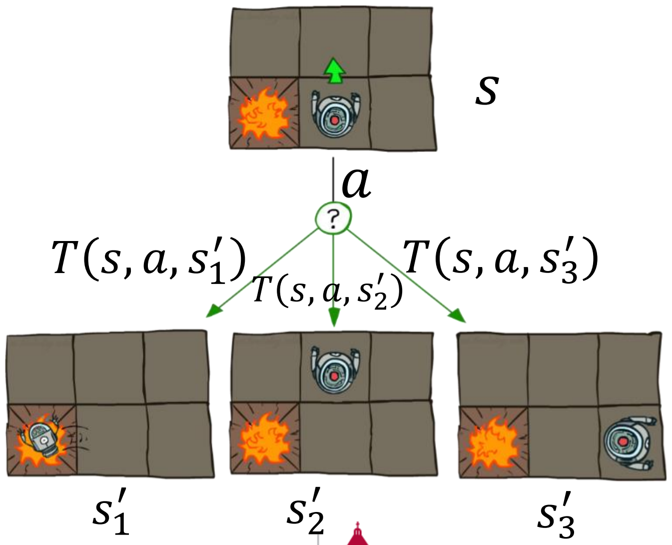
Markov Decision Processes

- $\sum_{s'} T(s,a,s') = 1$

  - For example
  - $\sum_{s'} T((1,1), North, s')$
    $= T((1,1), North, (1,2))$
    $+ T((1,1), North, (2,1))$
    $+ T((1,1), North, (1,1)) = 0.8 + 0.1 + 0.1$
    $= 1$

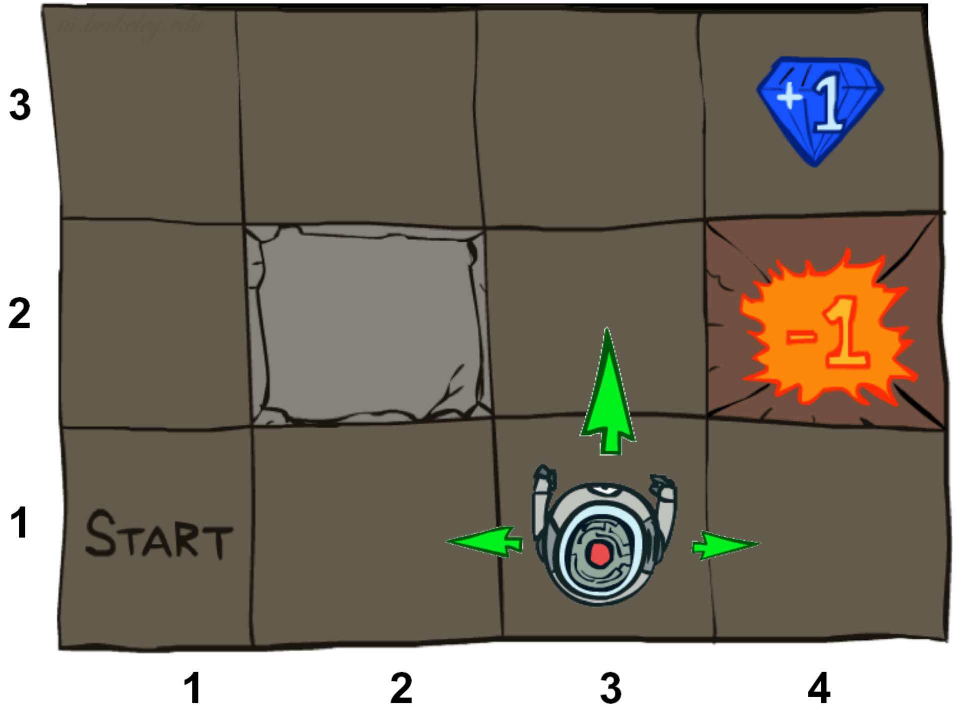
  - $\sum_{s'} T((3,1), North, s')$
    $= T((3,1), North, (3,2))$
    $+ T((3,1), North, (2,1))$
    $+ T((3,1), North, (4,1)) = 0.8 + 0.1 + 0.1$
    $= 1$





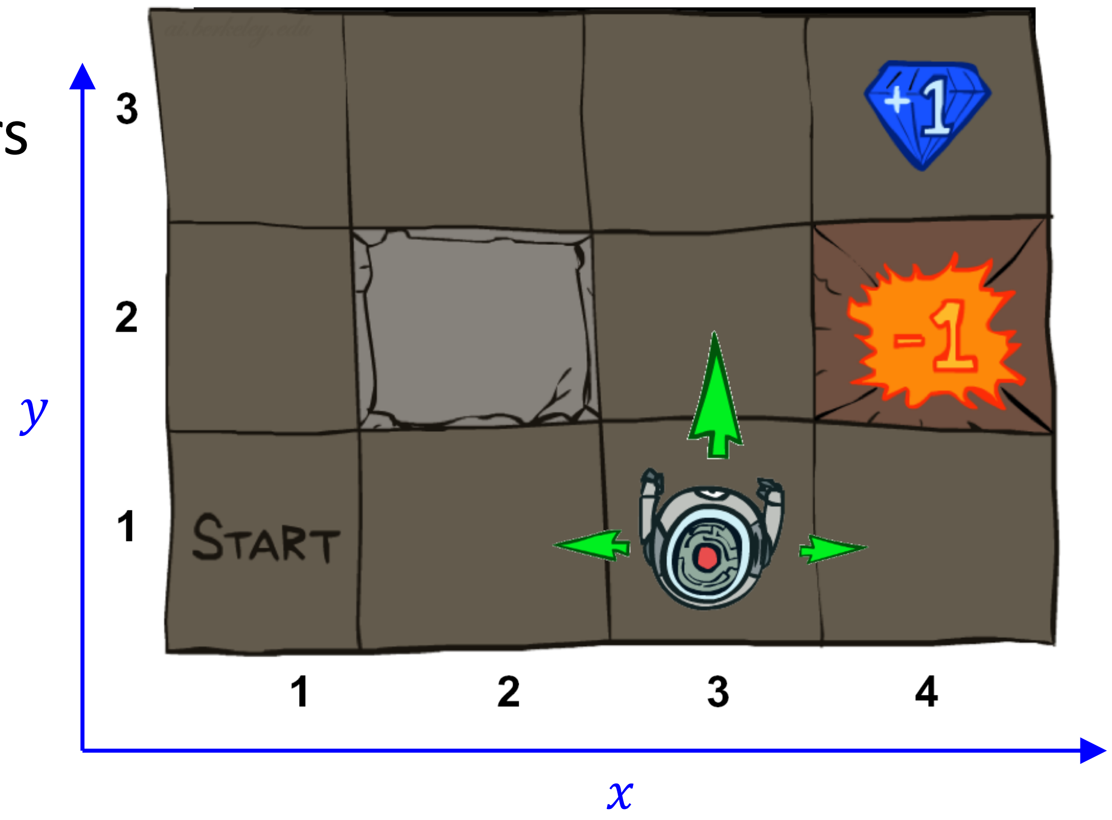
Markov Decision Processes

- **An MDP is defined by:**
  - A reward function
    - R(s,a,s')
    - R(s,a)
    - R(s)
  - A start state
  - Maybe a terminal state
- **MDPs are non-deterministic search problems**



Markov Decision Processes

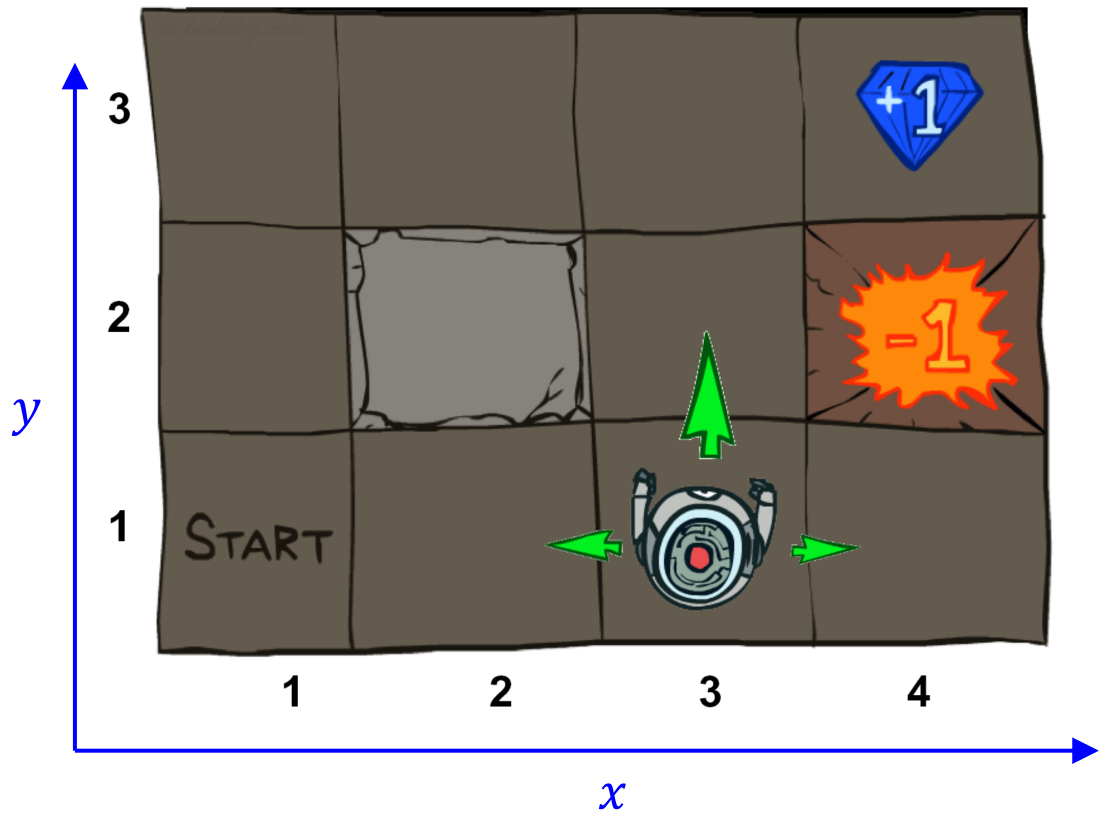
- **Example:**
- Assume the game is over if the agent enters (4,3) or (4,2).
- Assume a reward of +1 is gained when the agent enters (4,3); a reward of -1 is gained when the agent enters (4,2); for any other state transitions the reward is zero.
- What are the values of the following rewards?
  - When s=(3,3), a=East, s'=(4,3), R(s,a,s') =
  - When s=(3,2), a=East, s'=(4,2), R(s,a,s') =
  - When s=(4,1), a=North, s'=(4,2), R(s,a,s') =
  - When s=(4,1), a=North, s'=(3,1), R(s,a,s') =



Markov Decision Processes

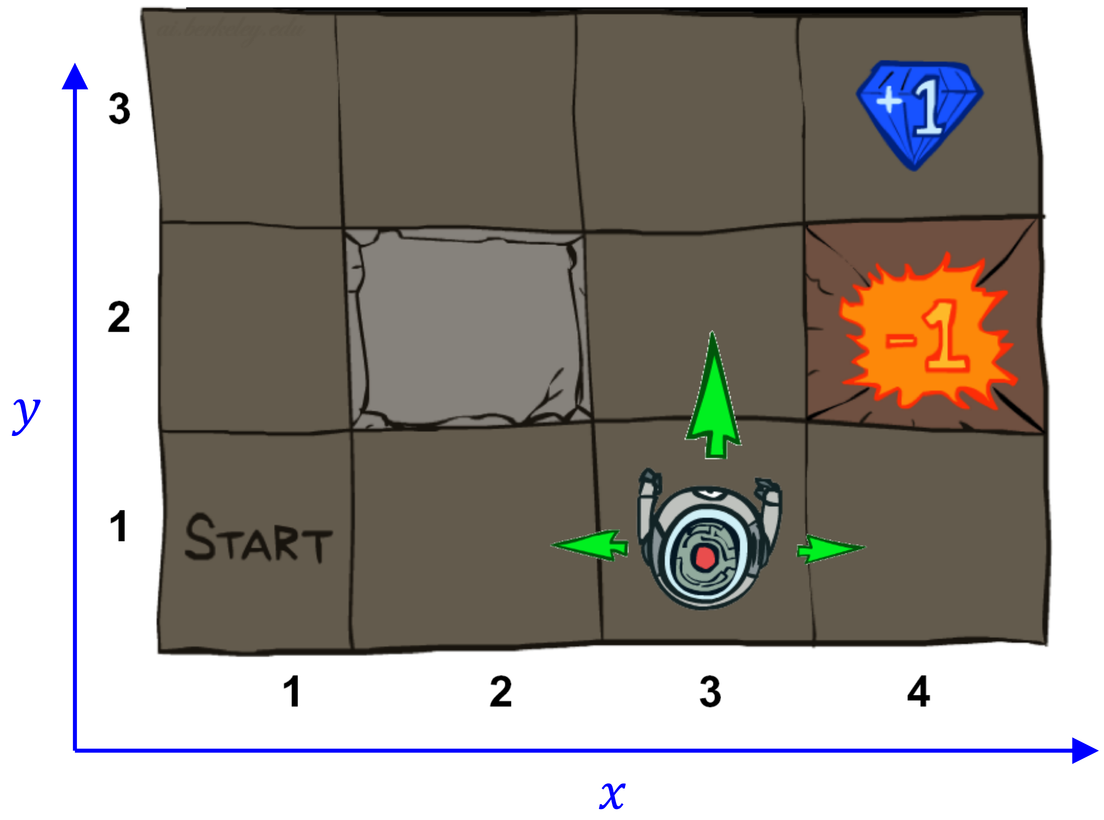
- **Example:**
  - Assume R(s,a,s’) is defined the same way as in the previous slide
  - Assume the agent is in (3,2)
  - He goes North
  - What’s the expected reward?

- **Answer: the reward expression is**
  - R((3,2),North,s’)
  - if s’ = (3,3), then R = 0. What’s the prob. of this situation? 0.8
  - if s’ = (4,2), then R = -1. What’s the prob. of this situation? 0.1
  - E[R] = 0.8*0 + 0.1*(-1)+0.1*0 = -0.1



Markov Decision Processes

- **Example:**
- If a reward of +1 is gained when the agent is in (4,3), and a reward of -1 is gained when the agent is in (4,2), and the reward is zero when the agent is in any other possible grid, then how to represent this kind of rewards?
  - R(s) = +1, when s = (4,3)
  - R(s) = -1, when s = (4,2)
  - R(s) = 0, for any other possible s

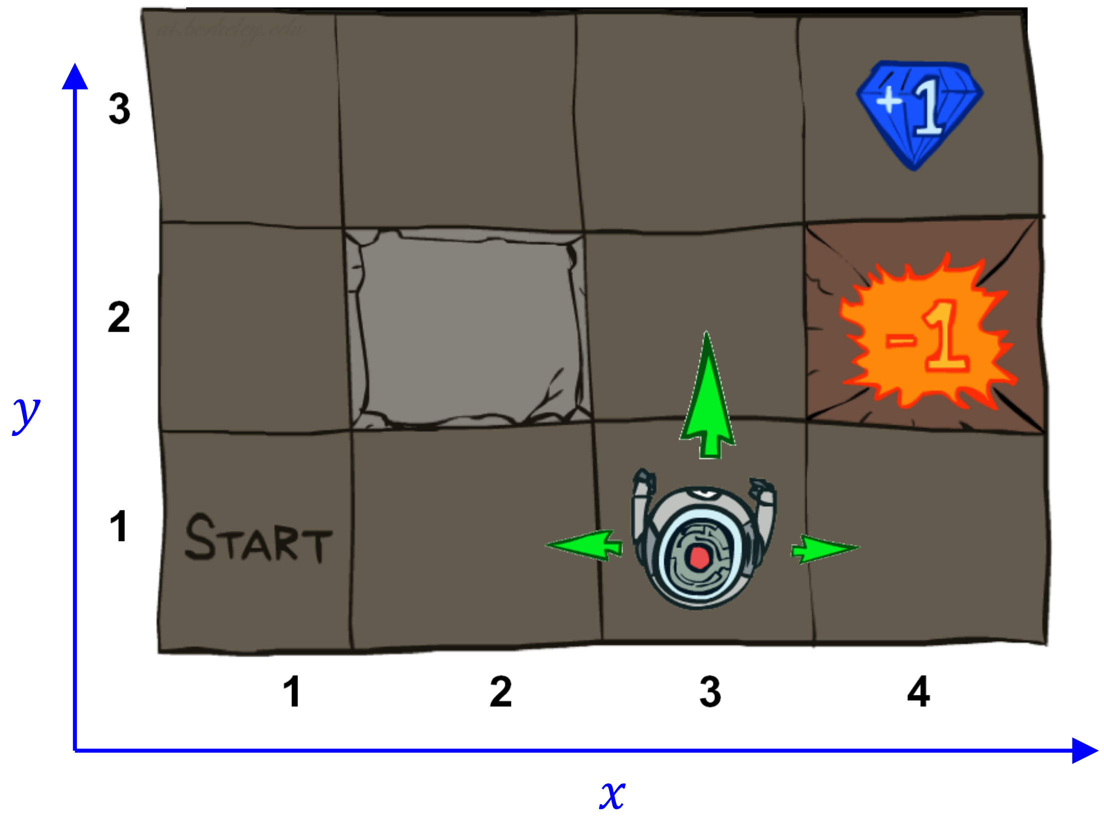


Markov Decision Processes

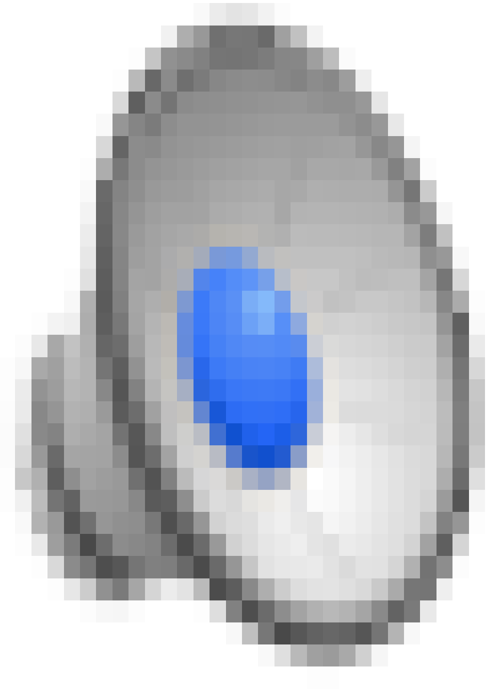
- Watch the video demo in the next slide.
- Think about how the reward function is defined in the video.

- R(s,a)
- R(s=(4,3), a=EXIT) = 1
- R(s=(4,2), a=EXIT) = -1

- R(s,a) = -0.1, for any other valid (s,a)
  - This is the living reward



Video Demo of Gridworld



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What is Markov about MDPs?

- **“Markov” generally means that given the present state, the future and the past are independent**

- **For Markov decision processes, “Markov” means action outcomes depend only on the current state**

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$

$$= \quad P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

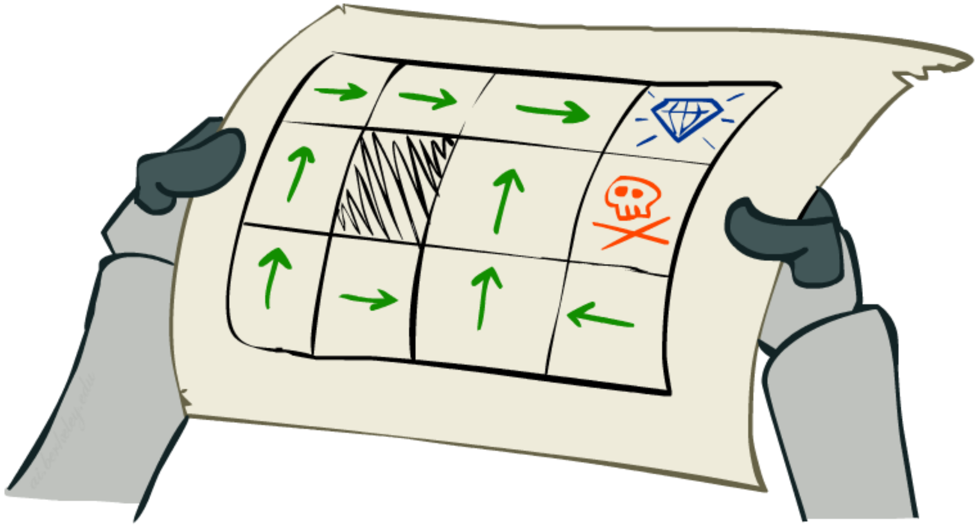
Andrey Markov
(1856-1922)

Policies

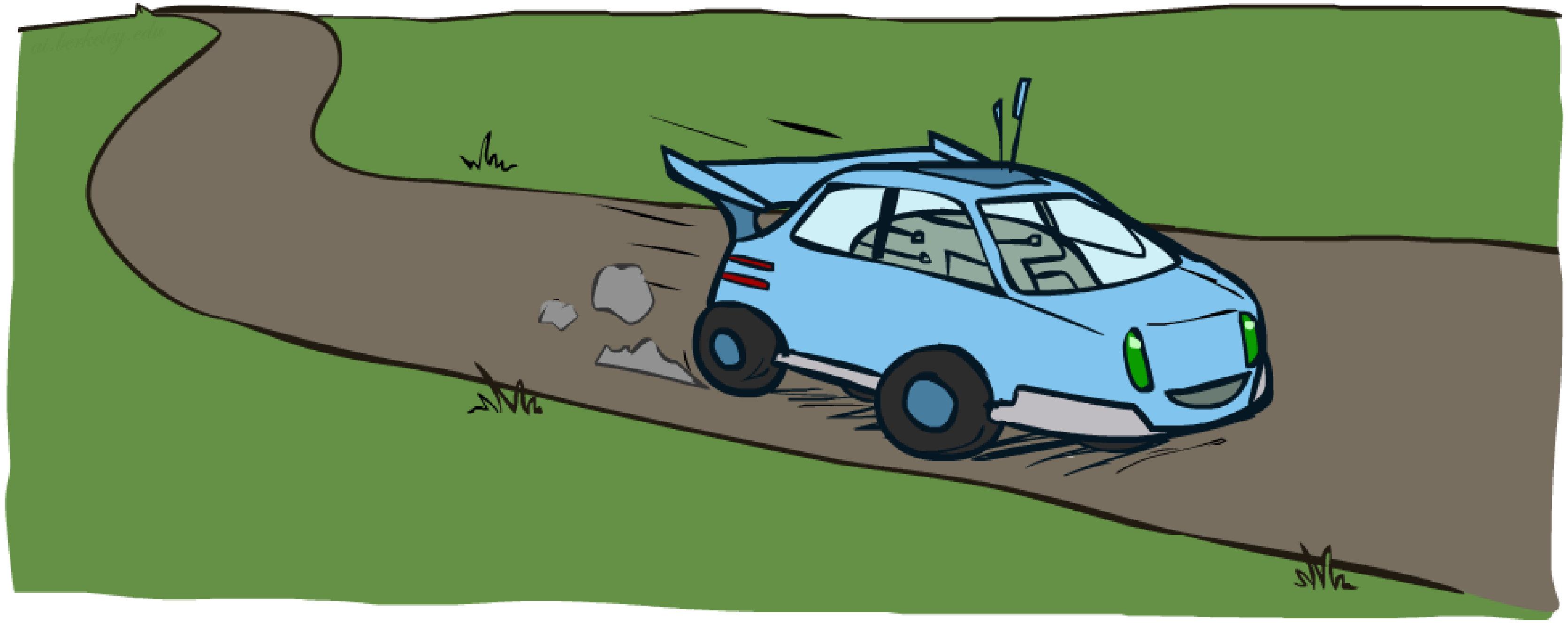
- In deterministic single-agent search problems, we wanted an optimal plan, or sequence of actions, from start to a goal

- For MDPs, we want an optimal policy $\pi^*: S \rightarrow A$

  - A policy π: a mapping from states to actions
  - A policy π gives an action for each state

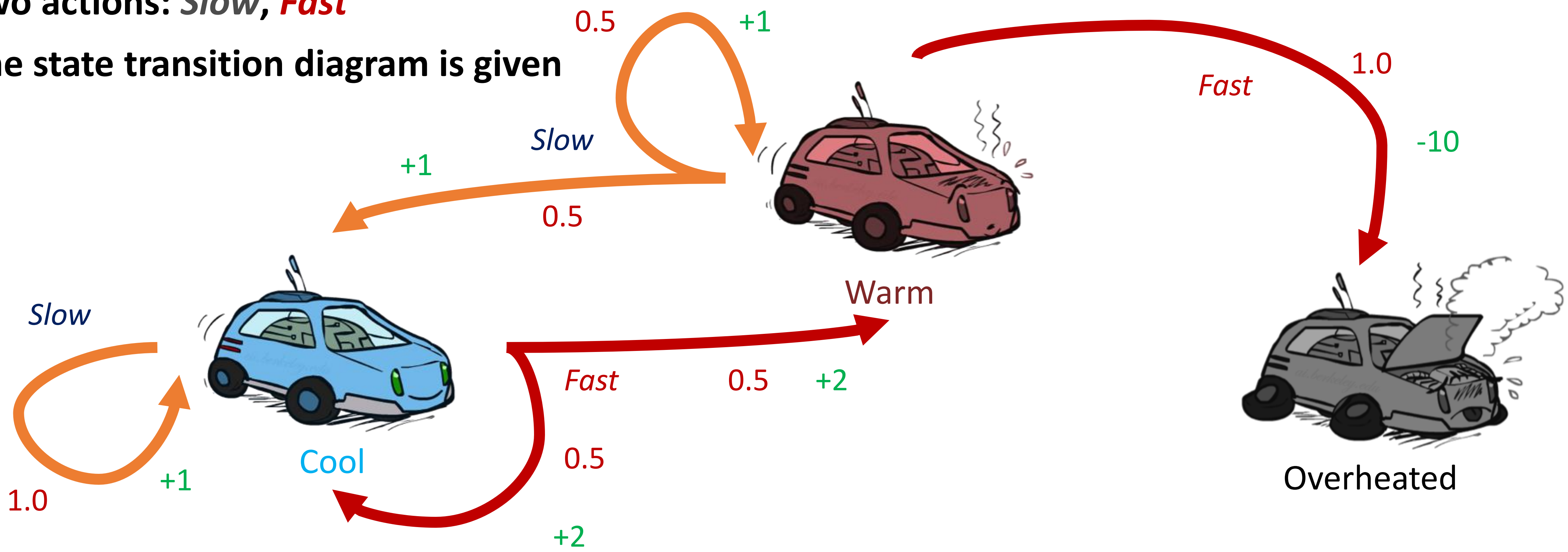


Example: Racing

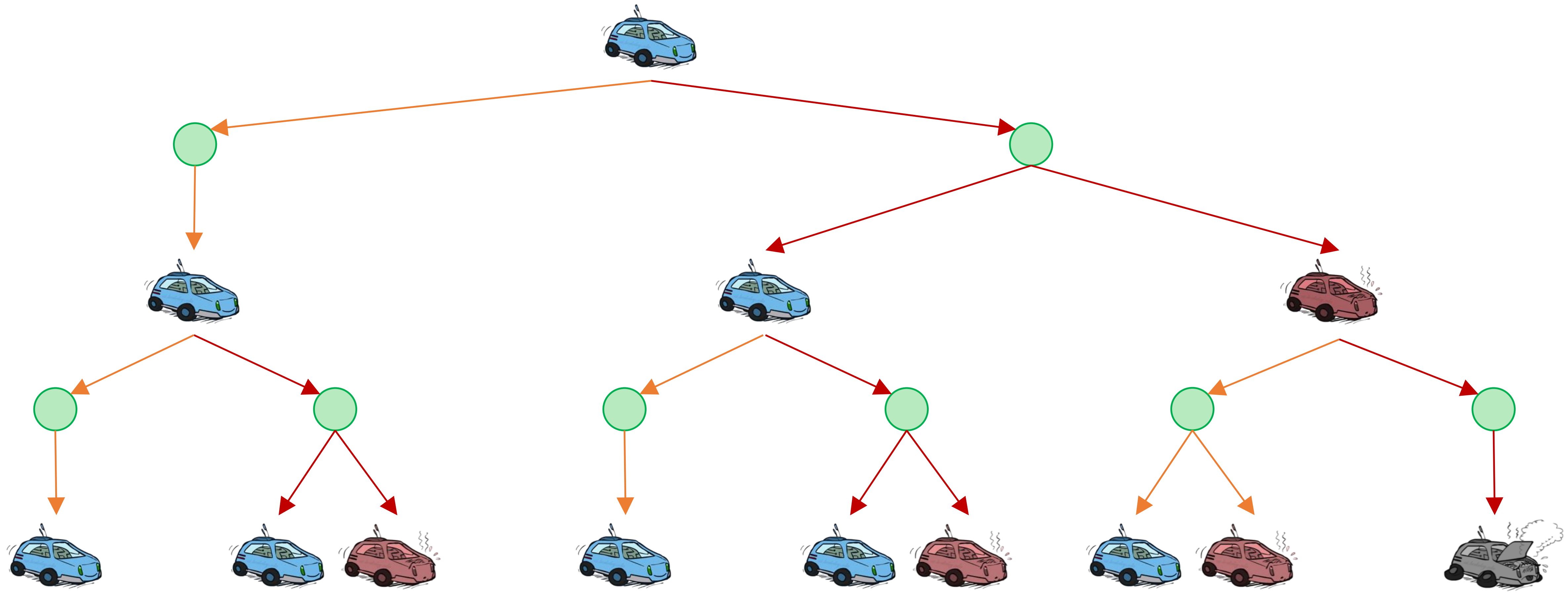


Example: Racing

- A robot car wants to travel far, quickly
- Three states: Cool, Warm, Overheated
- Two actions: *Slow*, ***Fast***
- The state transition diagram is given



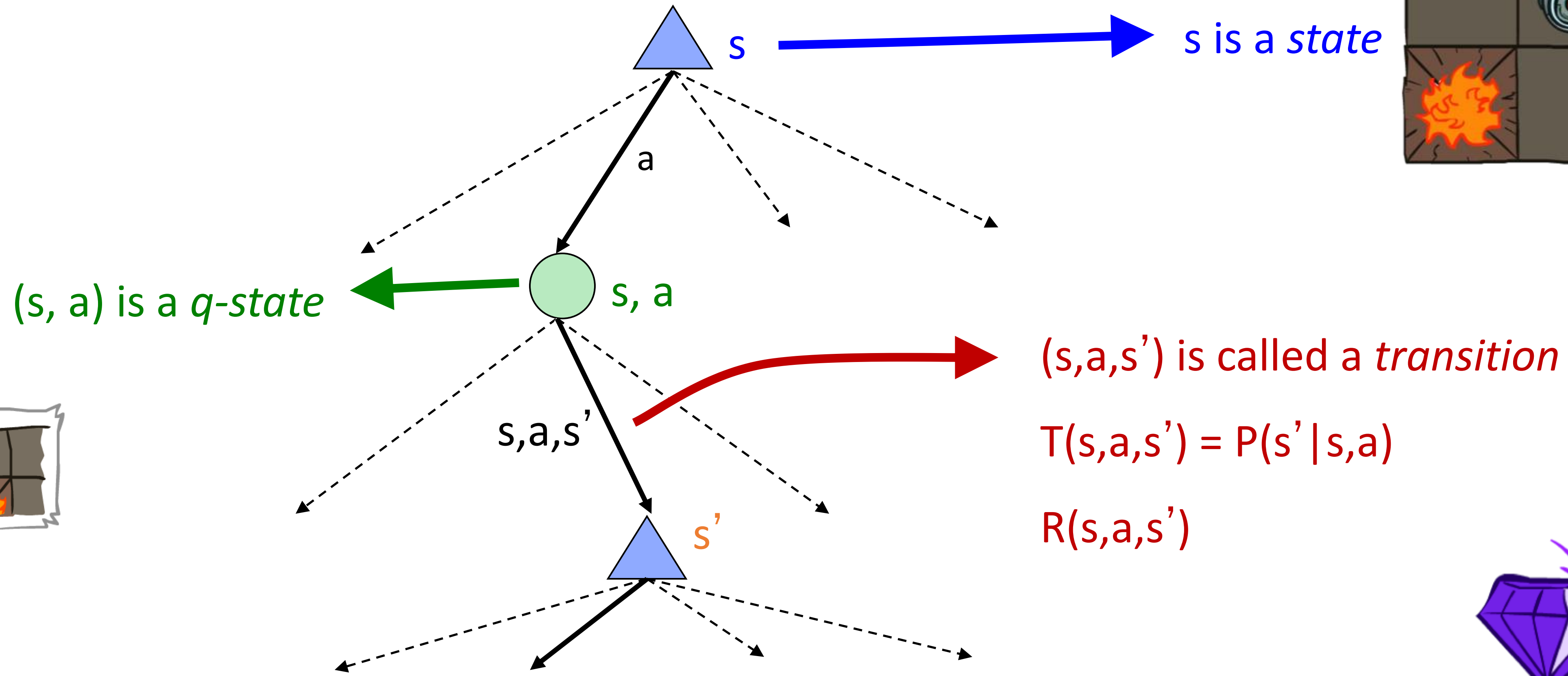
Racing Search Tree

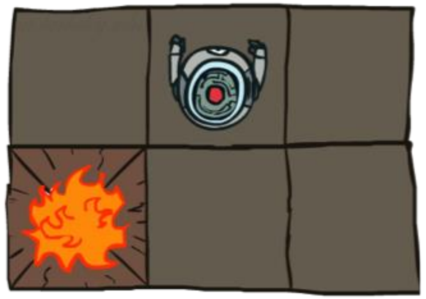


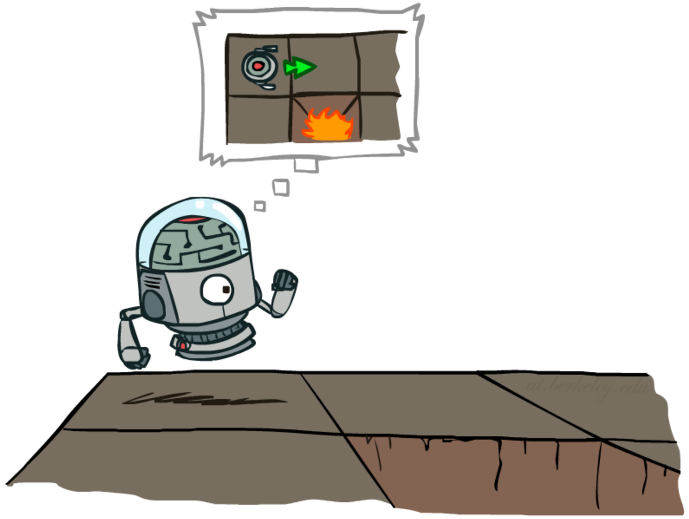
Can you label the branches with the action a, transition probability T(s,a,s’), and reward R(s,a,s’)?

MDP Search Trees

- **Q-state: (s,a)**

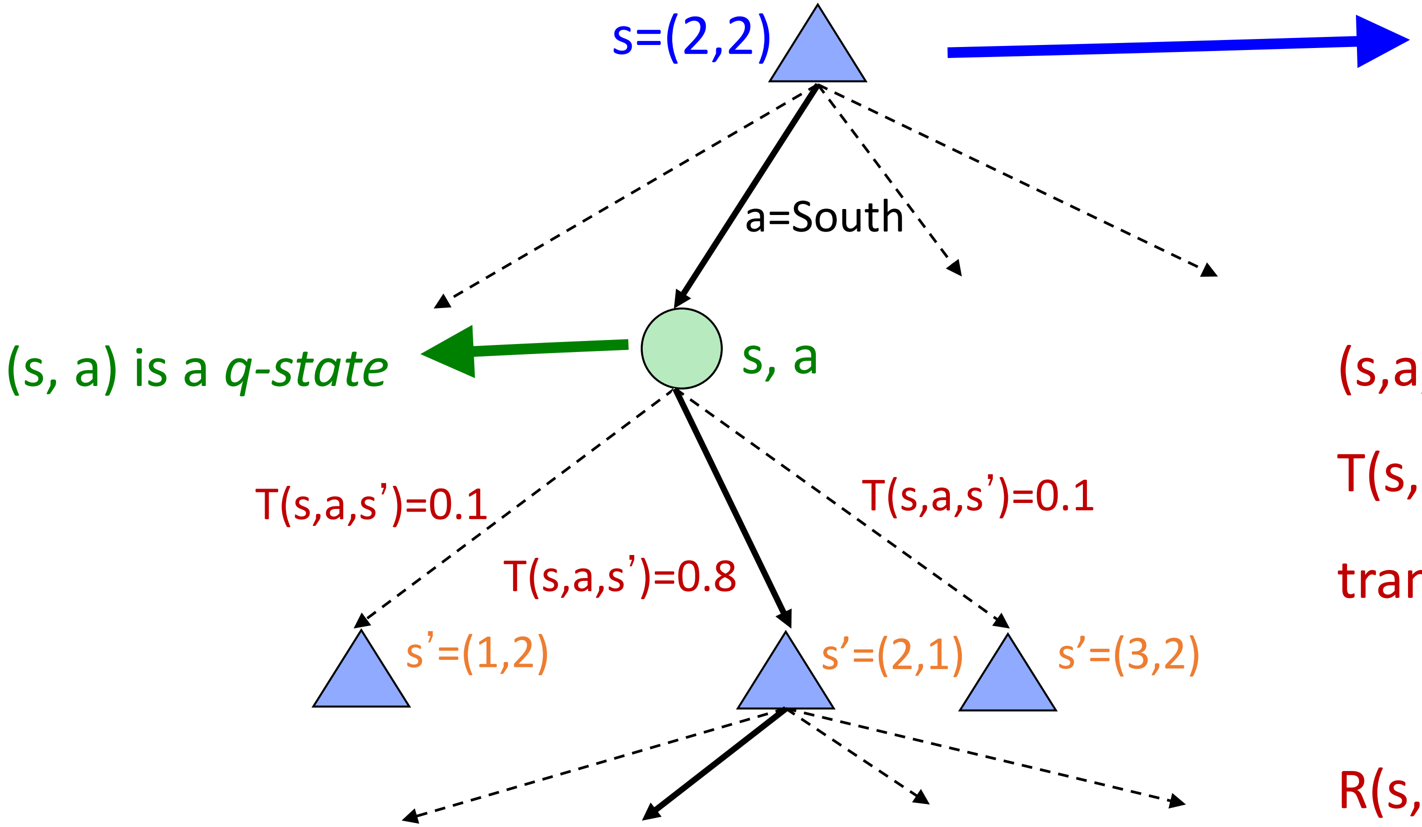




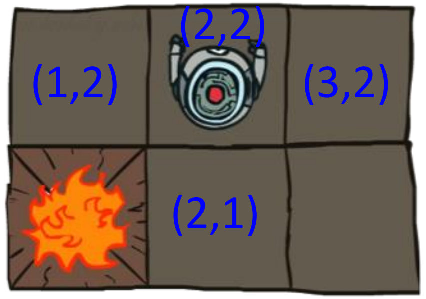


MDP Search Trees

- **Q-state: (s,a)**



s is a *state*



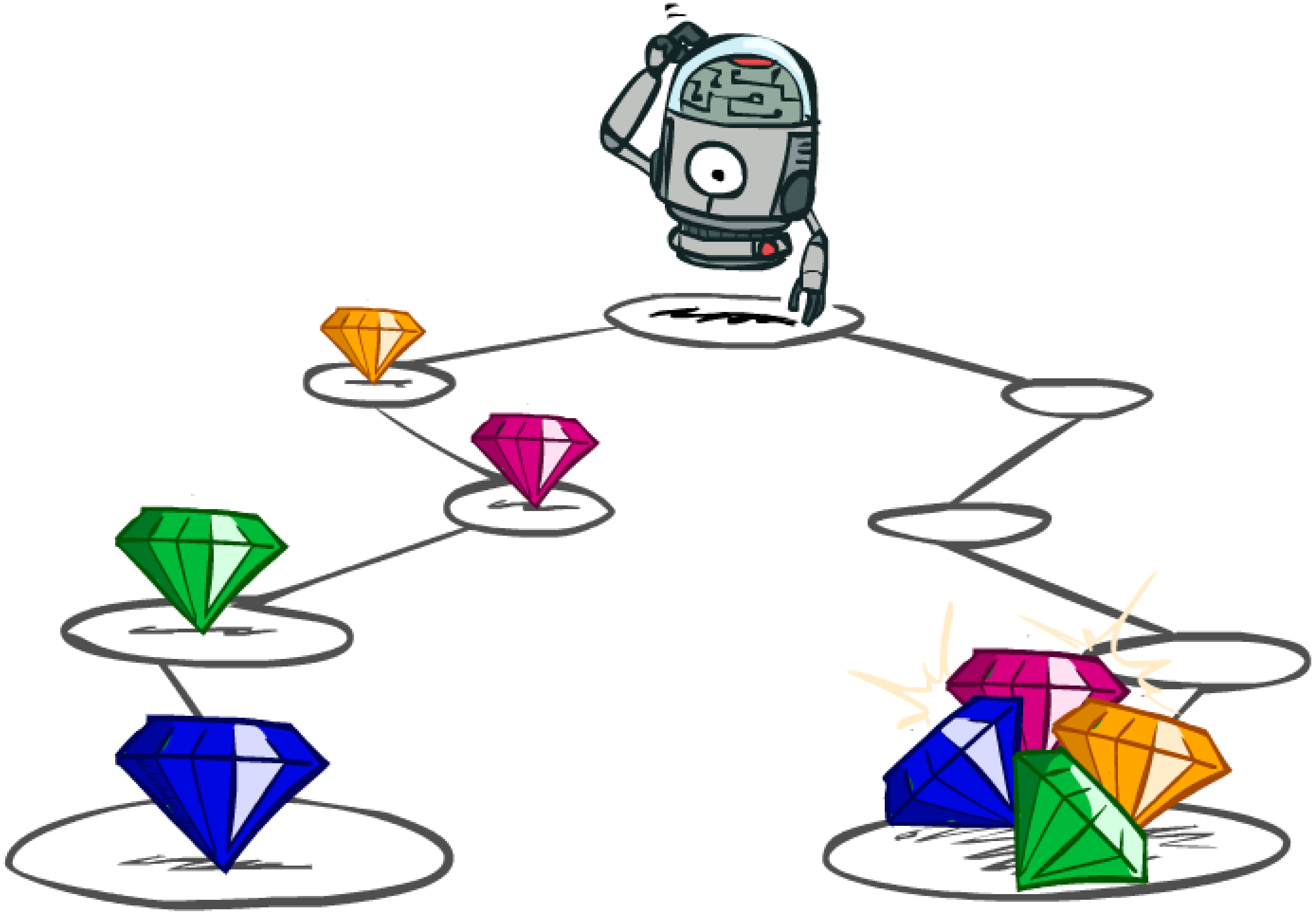
(s,a,s') called a *transition*

T(s,a,s') = P(s'|s,a):

transition probability

R(s,a,s'): (immediate) reward

Reward Sequences



Reward Sequences

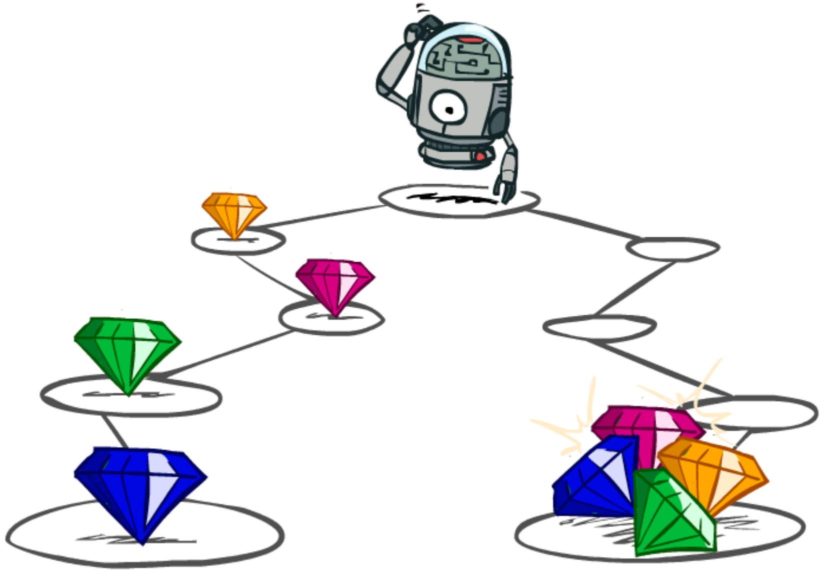
- **What preferences should an agent have over reward sequences?**

- **More or less?**

  [1, 2, 2] or [2, 3, 4]

- **Now or later?**

  [0, 0, 1] or [1, 0, 0]



Discounting

- It’s reasonable to let the agent take actions to maximize the sum of rewards
- It’s also reasonable to prefer rewards now to rewards later
- One solution: values of rewards decay exponentially

1

Worth Now

γ

Worth Next Step

γ^2

Worth In Two Steps

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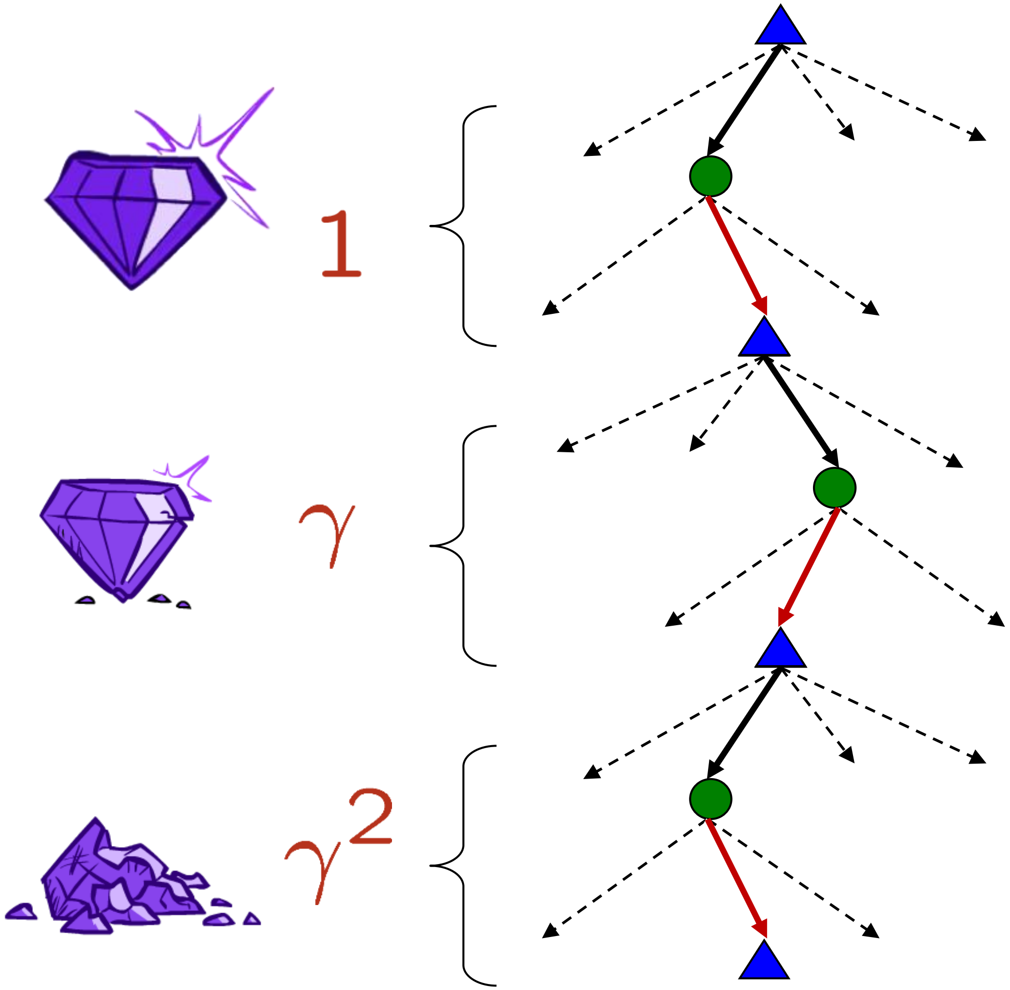
Discounting

- **How to discount?**
  - Each time we descend a level, we multiply in the discount once

- **Why discount?**
  - Sooner rewards probably have higher utility than later rewards

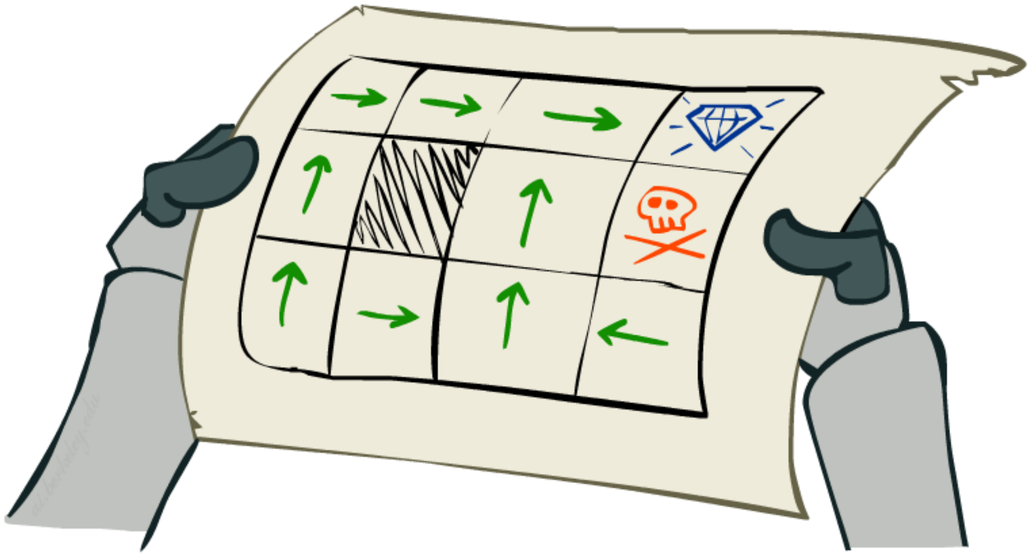


Solving MDPs

- **How to figure out the optimal policy π^* for the agent?**

- **That is, the best action at each state:**

  $\pi^*(s)\ \forall s \in S$

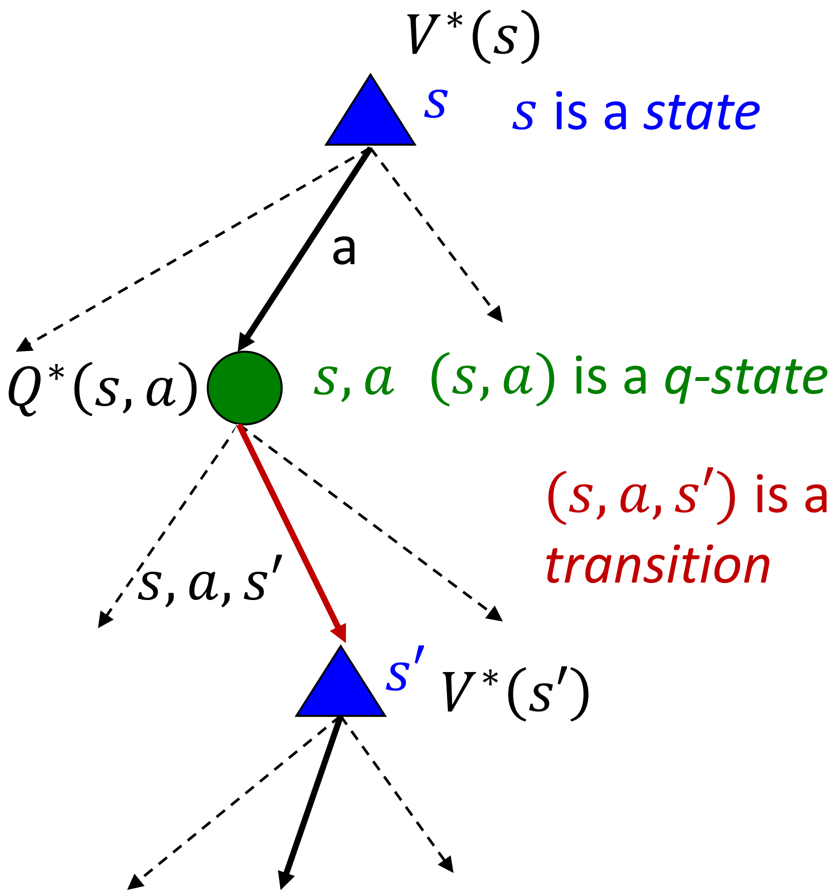


Optimal Quantities

- **The value (utility) of a state s:**
  $V^*(s)$ = expected value/utility starting in s and acting optimally

- **The value (utility) of a q-state (s,a):**
  $Q^*(s,a)$ = expected value/utility starting out having taken action a from state s and (thereafter) acting optimally

- **The optimal policy:**
  $\pi^*(s)$ = optimal action at state s



Values of States

- **Values of states**

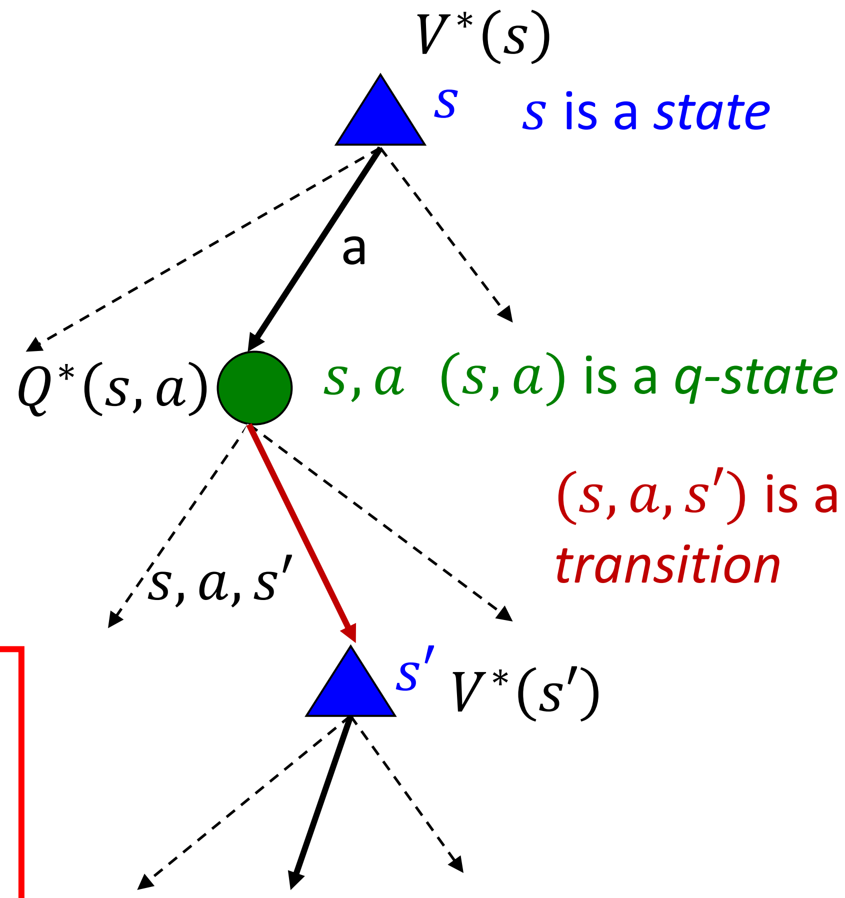
  $$V^*(s) = \max_a Q^*(s,a)$$

- **Q-state value**

  $$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

- **Recursive definition of value:**
- **Bellman Equation**

  $$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$



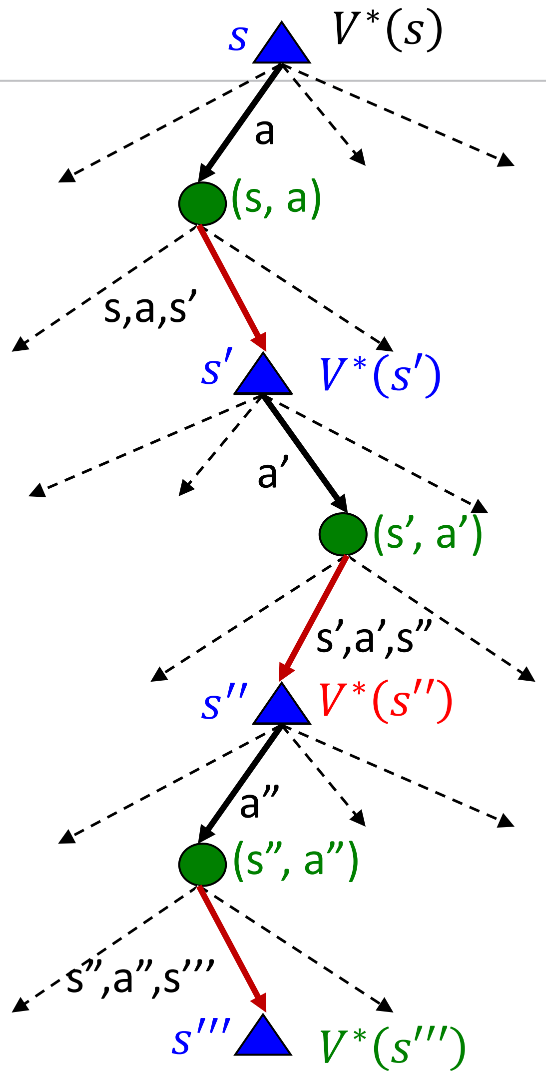
Bellman Equation

- $$V^*(s) = \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

$$\max_{a'} \sum_{s''} T(s',a',s'')[R(s',a',s'') + \gamma V^*(s'')]$$

$$\max_{a''} \sum_{s'''} T(s'',a'',s''')[R(s'',a'',s''') + \gamma V^*(s''')]$$

…



Bellman Equation

- $V^*(s) = \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$

- To simplify the problem, let's assume the actions are fixed, and no randomness:

$$V^*(s) = R(s,a,s') + \gamma V^*(s')$$

$$\uparrow$$

$$\boxed{R(s',a',s'') + \gamma V^*(s'')}$$

$$\uparrow$$

$$\boxed{R(s'',a'',s''') + \gamma V^*(s''')}$$

$$V^*(s) = R(s,a,s') + \gamma R(s',a',s'') + \gamma^2 R(s'',a'',s''') + \cdots$$

$V^*(s)$ is the sum of discounted rewards.

Bellman Equation

- **When there is no randomness, and the actions are fixed:**
  $V^*(s)$ is the sum of discounted rewards.

- **In general, there is randomness, and the agent needs to consider multiple action choices, hence**

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \times [R(s,a,s') + \gamma V^*(s')]$$

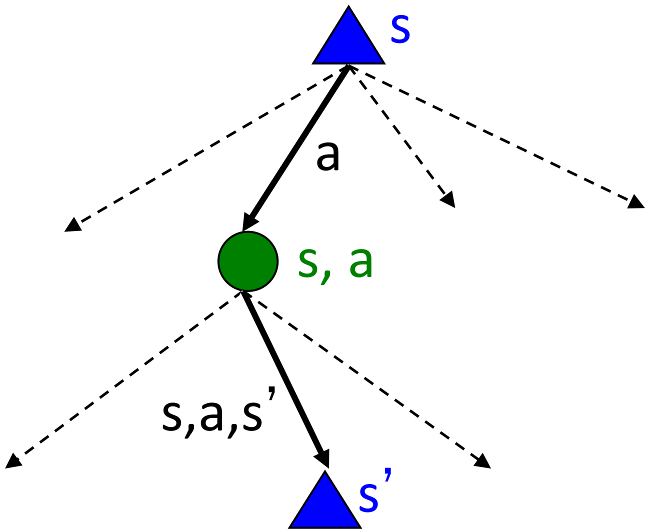
$V^*(s)$ = **Max (Expectation of (the sum of discounted rewards))**

- **Recall: V*(s) = expected value/utility starting in s and acting optimally**

How to calculate $V^*(s)$?

- **Recursive definition of value:**
  - **Bellman Equation**

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$



$$V^*(s_1) = f_1\big(V^*(s_1), V^*(s_2), \dots, V^*(s_N)\big)$$
$$V^*(s_2) = f_2\big(V^*(s_1), V^*(s_2), \dots, V^*(s_N)\big)$$
$$\vdots$$
$$V^*(s_N) = f_N\big(V^*(s_1), V^*(s_2), \dots, V^*(s_N)\big)$$

Assume there are N states, then
we have N unknowns and N nonlinear equations

Linear Equation

- Two unknowns x and y
- Two equations

$$c_1 = a_1 x + b_1 y$$
$$c_2 = a_2 x + b_2 y$$

- How to solve for x and y?

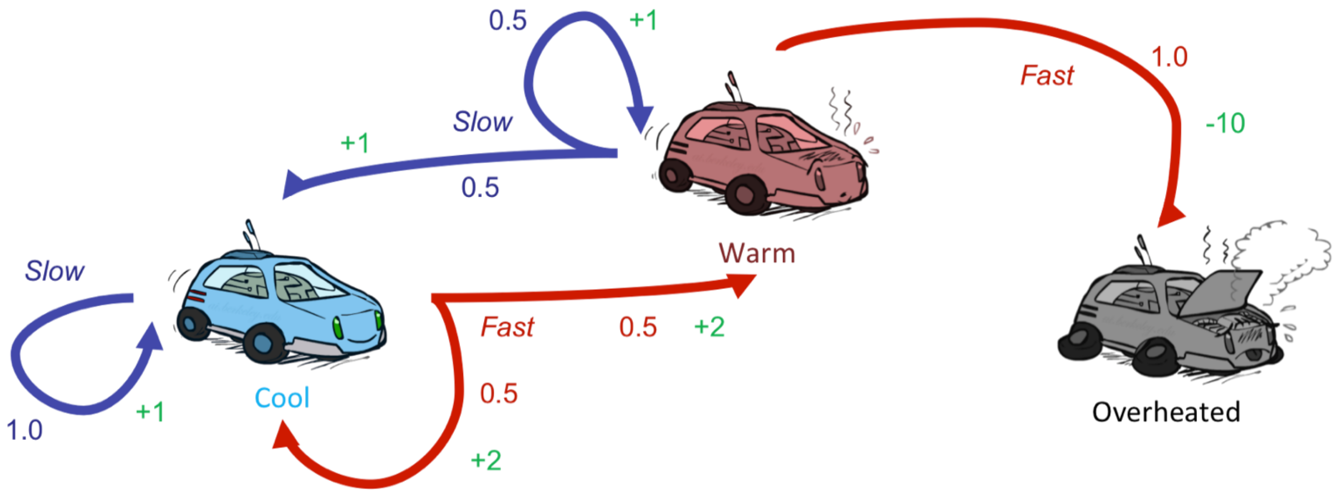
$$x = \frac{c_1 - b_1 y}{a_1} \quad \dots (1)$$

$$c_2 = a_2 \frac{c_1 - b_1 y}{a_1} + b_2 y$$

$\Rightarrow$ Solve for y, then plug y into (1) to solve for x

Value Iteration

V_2	3.5	2.5	0
V_1	2	1	0
V_0	0	0	0



Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

Value Iteration

- **Step 1:** Initialize $V_0(s) = 0$, for $s = s_1, s_2, s_3, \ldots$
- **Step 2:** k=1 (1^{st} iteration)

  Update $V_1(s)$, for $s = s_1, s_2, s_3, \ldots,$

  using $V_0(s) = 0$, $s = s_1, s_2, s_3, \ldots$

  $$V_1(s) \leftarrow \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V_0(s')]$$

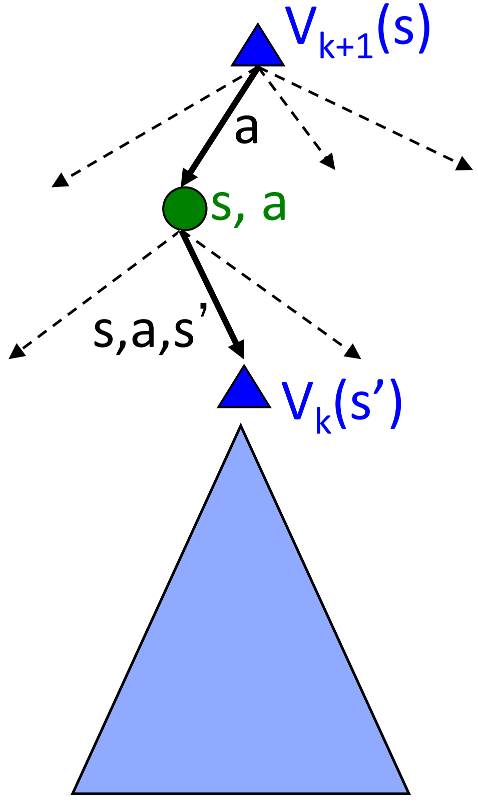
- **Step 3:** k=2 (2^{nd} iteration)

  Update $V_2(s)$, for $s = s_1, s_2, s_3, \ldots,$

  using $V_1(s)$, $s = s_1, s_2, s_3, \ldots$

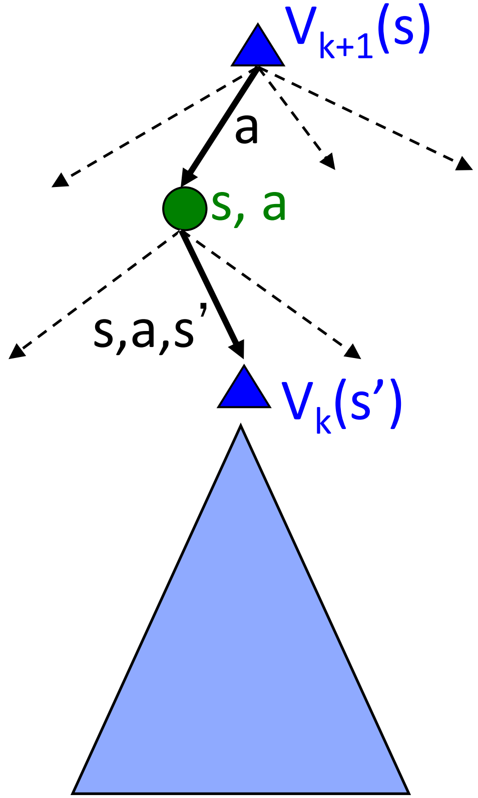
  $$V_2(s) \leftarrow \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V_1(s')]$$

- **Keep running the iterations till the values $V_k(s)$ converge.**



Value Iteration

- **Start with $V_0(s) = 0$**
- **Given vector of $V_k(s)$ values, do one ply of expectimax from each state:**



- **Bellman Update**

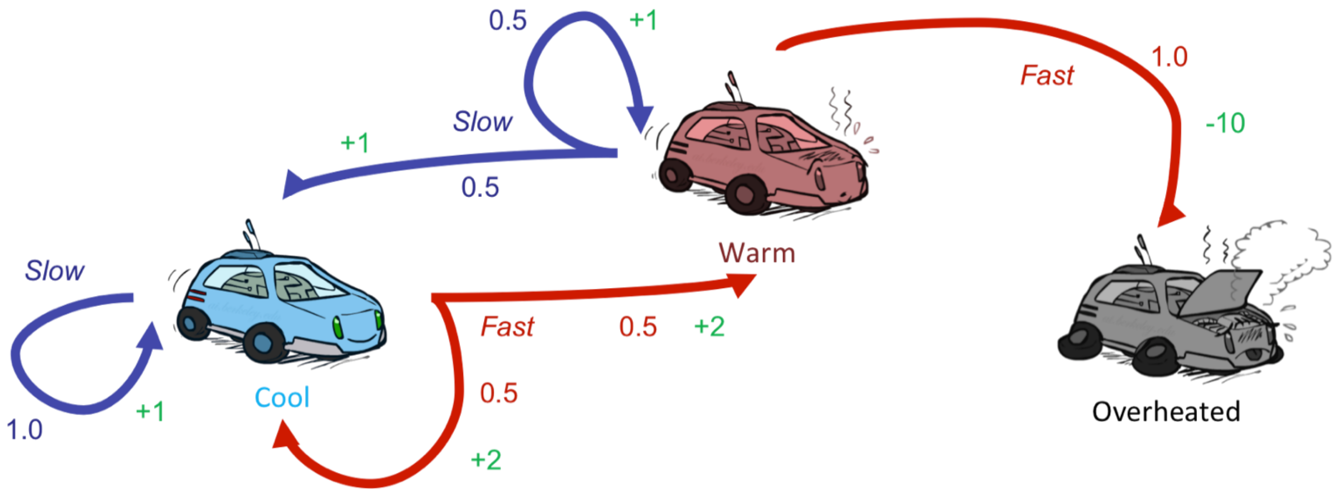
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

- **Repeat until convergence**

- **Complexity of each iteration:** $O(|S|^2 \times |A|)$
  - $|S|$: The cardinality of the set S, i.e. the number of elements in the set S

Example: Value Iteration

V_2	3.5	2.5	0
V_1	2	1	0
V_0	0	0	0



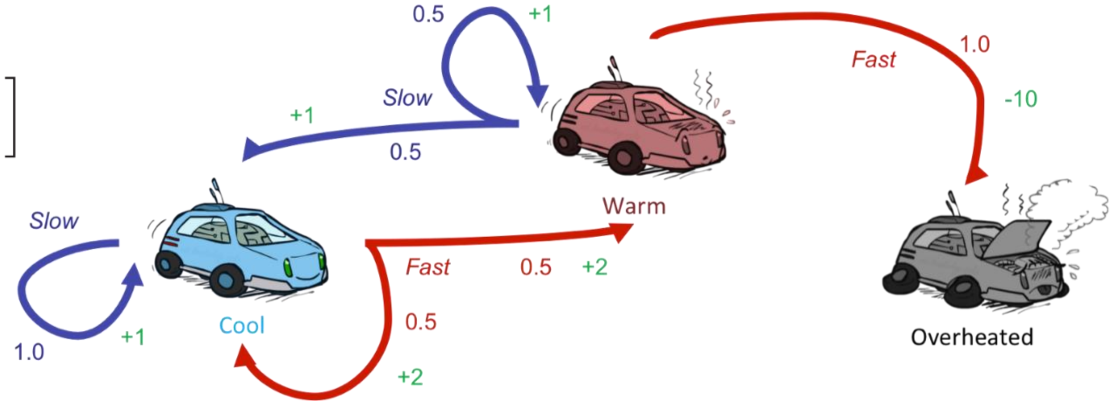
Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

$V_0(cool) = 0,\ \ V_0\ (warm) = 0,\ \ V_0(overheated) = 0$

- $V_1(cool)$:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$



- **1. a=slow**
  - $T(s = cool, a = slow, s' = cool) = 1$

    $R(cool, slow, s' = cool) = 1$
  - $V_0(s' = cool) = 0$

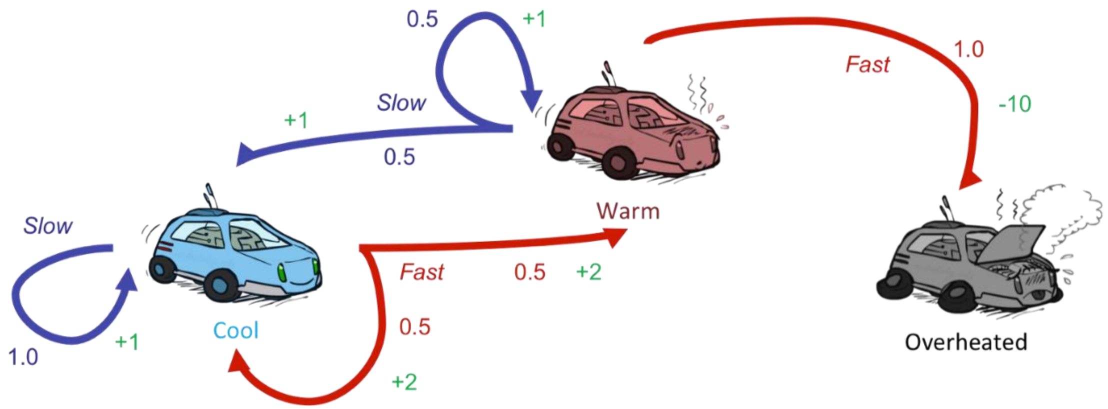
  - $Result = T(cool, slow, s' = cool) \times [R(cool, slow, s' = cool) + \gamma \times V_0(s' = cool)]$
    $= 1 \times [1 + 1 \times 0] = 1$

$$V_0(cool) = 0,\ \ V_0\ (warm) = 0,\ \ V_0(overheated) = 0$$

- $V_1(cool)$:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- **2. a=fast**
  - $T(cool, fast, s' = cool) = 0.5$
    $R(cool, fast, s' = cool) = 2$
  - $T(cool, fast, s' = warm) = 0.5$
    $R(cool, fast, s' = warm) = 2$



- $Result\ = T(cool, fast, s' = cool) \times [R(cool, fast, s' = cool) + \gamma \times V_0(s' = cool)]$
$+ T(cool, fast, s' = warm) \times [R(cool, fast, s' = warm) + \gamma \times V_0(s' = warm)] = 2$

Hence, $V_1(cool) = 2$

$V_0(cool) = 0,\ \ V_0\ (warm) = 0,\ \ V_0(overheated) = 0$

- $V_1(warm)$:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- **1. a=slow**

$$T(s = warm, a = slow, s' = cool) = 0.5$$

$$R(warm, slow, s' = cool) = 1$$

$$T(warm, slow, s' = warm) = 0.5$$

$$R(warm, slow, s' = warm) = 1$$

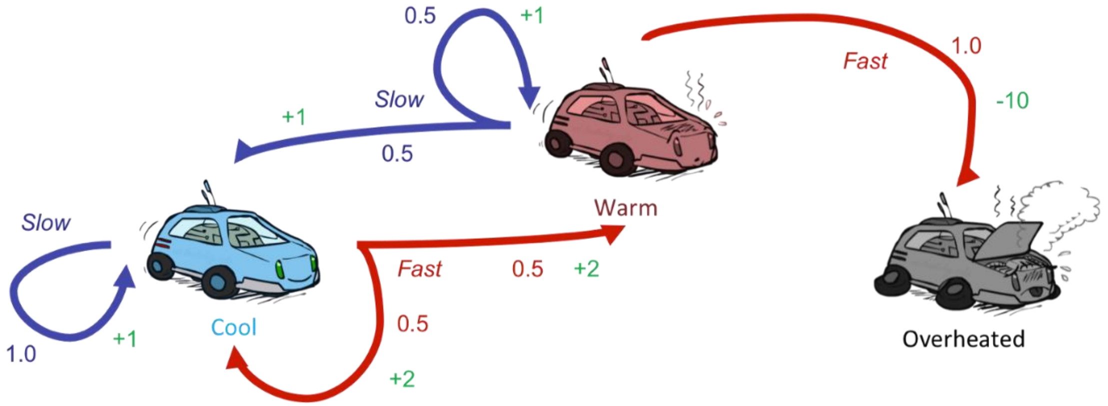
- $Result\ = T(warm, slow, s' = cool) \times [R(warm, slow, s' = cool) + \gamma \times V_0(s' = cool)]$

$+ T(warm, slow, s' = warm) \times [R(warm, slow, s' = warm) + \gamma \times V_0(s' = warm)] = 1$

$V_0(cool) = 0,\ V_0\ (warm) = 0,\ V_0(overheated) = 0$

- $V_1(warm)$:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$



- **2. a=fast**
  - $T(warm, fast, s' = overheated) = 1$

    $R(warm, fast, s' = overheated) = -10$
  - $V_0(s' = overheated) = 0$

  - $Result = T(warm, fast, s' = overheated) \times [R(warm, fast, s' = overheated) + \gamma \times V_0(s' = overheated)]$

    $= 1 \times [-10 + 1 \times 0] = -10$

Hence, $V_1(warm) = 1$

$V_0(cool) = 0,\ V_0\ (warm) = 0,\ V_0(overheated) = 0$

- $V_1(overheated)$:

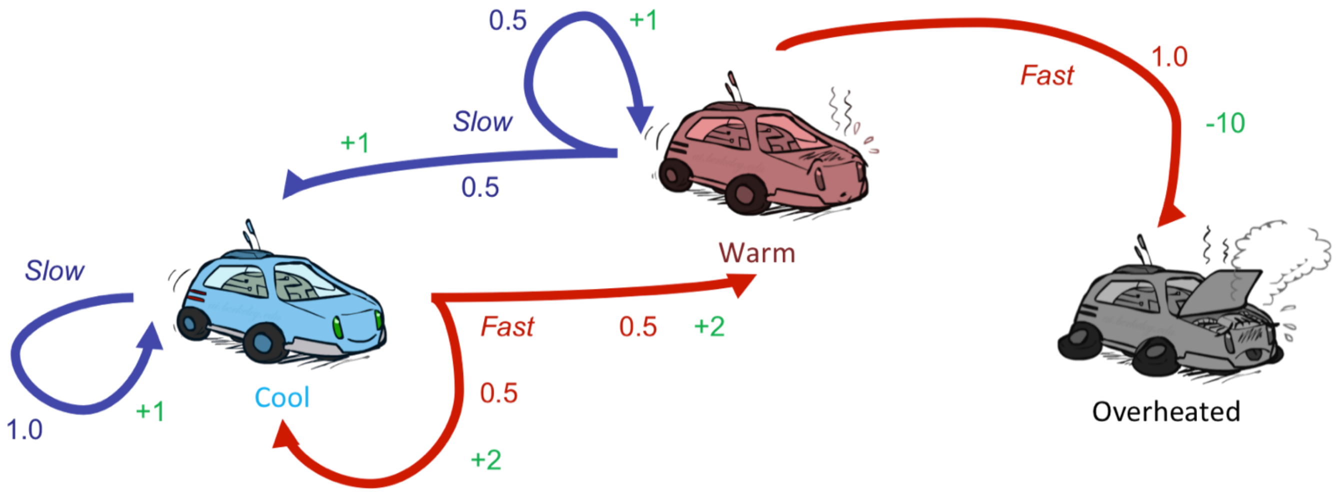
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- **Overheated is the end state**
  - No more state transition (no s')
  - No $T(overheated, a, s$'), no $R(overheated, a, s$')

Hence, $V_1(overheated) = 0$

Example: Value Iteration

V_2	3.5	2.5	0
V_1	2	1	0
V_0	0	0	0



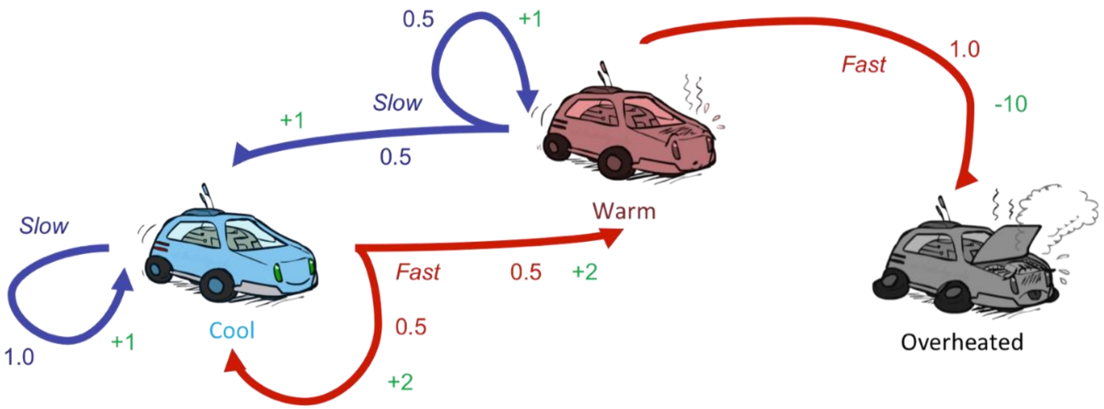
Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

$V_1(cool) = 2,\ \ V_1\ (warm) = 1,\ \ V_1(overheated) = 0$

- $V_2(cool)$:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$



- **1. a=slow**
  - $T(cool, slow, s' = cool) = 1$
    $R(cool, slow, s' = cool) = 1$
  - $V_1(s' = cool) = 2$

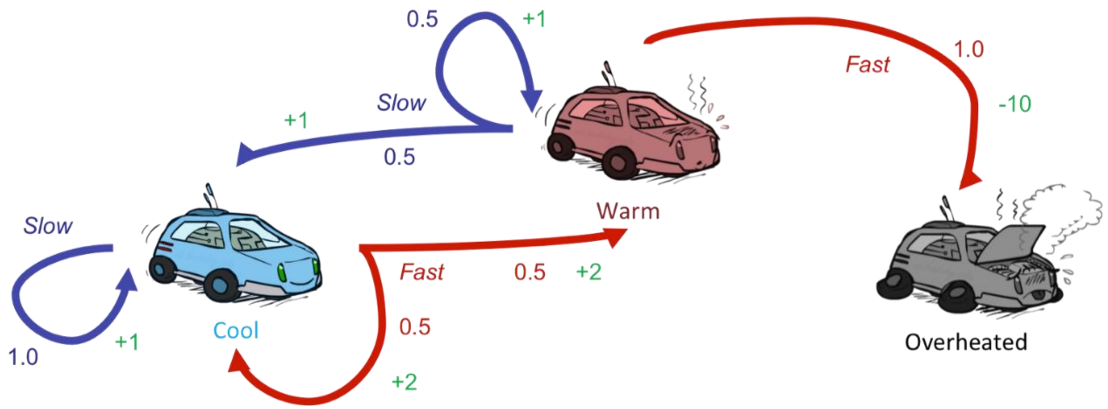
  - $Result = T(cool, slow, s' = cool) \times [R(cool, slow, s' = cool) + \gamma \times V_1(s' = cool)]$
    $= 1 \times [1 + 1 \times 2] = 3$

$V_1(cool) = 2, \; V_1\,(warm) = 1, \; V_1(overheated) = 0$

- $V_2(cool)$:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- **2. a=fast**
  - $T(cool, fast, s' = cool) = 0.5$
    $R(cool, fast, s' = cool) = 2$
  - $T(cool, fast, s' = warm) = 0.5$
    $R(cool, fast, s' = warm) = 2$



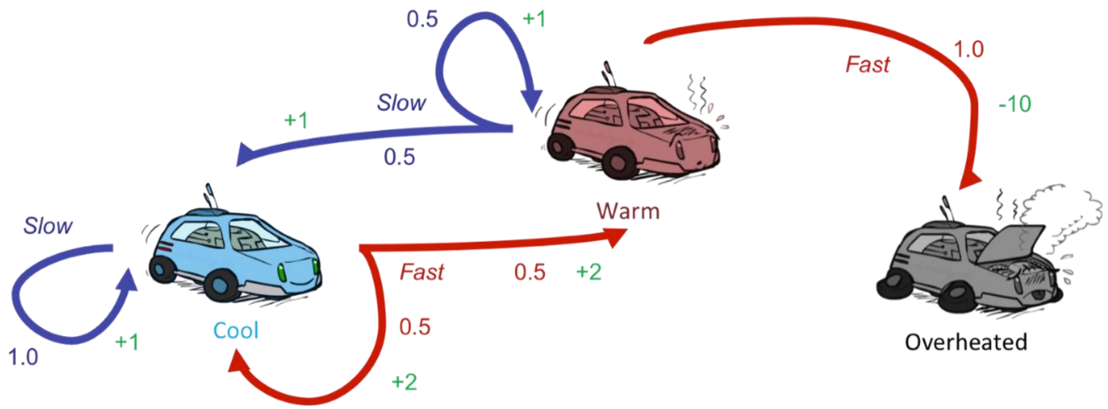
- $Result = T(cool, fast, s' = cool) \times [R(cool, fast, s' = cool) + \gamma \times V_1(s' = cool)]$
  $+ T(cool, fast, s' = warm) \times [R(cool, fast, s' = warm) + \gamma \times V_1(s' = warm)]$
  $= 0.5 \times [2 + 1 \times 2] + 0.5 \times [2 + 1 \times 1] = 2 + 1.5 = 3.5$

Hence, $V_2(cool) = 3.5$

$V_1(cool) = 2,\ V_1\ (warm) = 1,\ V_1(overheated) = 0$

- $V_2(warm)$:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$



- **1. a=slow**
  - $T(warm, slow, s' = cool) = 0.5$
    $R(warm, slow, s' = cool) = 1$
  - $T(warm, slow, s' = warm) = 0.5$
    $R(warm, slow, s' = warm) = 1$

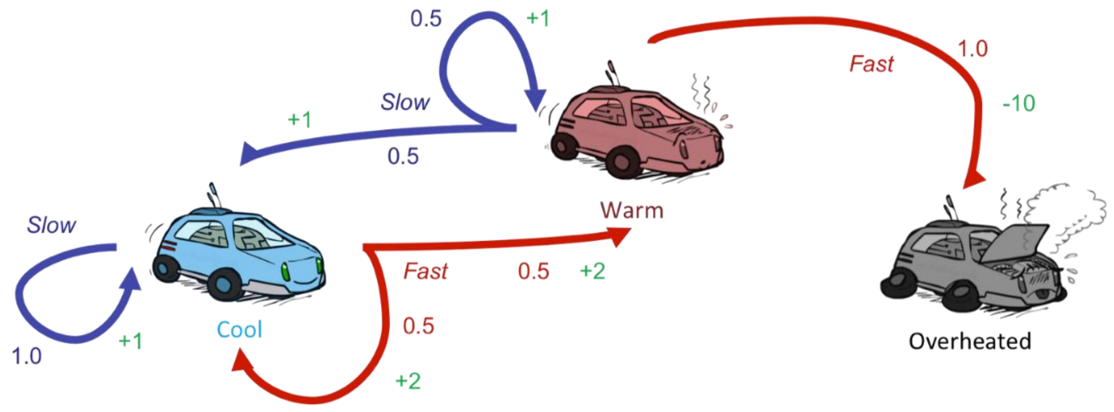
- $Result\ = T(warm, slow, s' = cool) \times [R(warm, slow, s' = cool) + \gamma \times V_1(s' = cool)]$
$+ T(warm, slow, s' = warm) \times [R(warm, slow, s' = warm) + \gamma \times V_1(s' = warm)]$

$$= 0.5 \times [1 + 1 \times 2] + 0.5 \times [1 + 1 \times 1] = 1.5 + 1 = 2.5$$

$V_1(cool) = 2,\ V_1\ (warm) = 1,\ V_1(overheated) = 0$

- $V_2(warm)$:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$



- **2. a=fast**
  - $T(warm, fast, s' = overheated) = 1$

    $R(warm, fast, s' = overheated) = -10$
  - $V_1(s' = overheated) = 0$

  - $Result = T(warm, fast, s' = overheated) \times [R(warm, fast, s' = overheated) + \gamma \times V_1(s' = overheated)]$

  $= 1 \times [-10 + 1 \times 0] = -10$

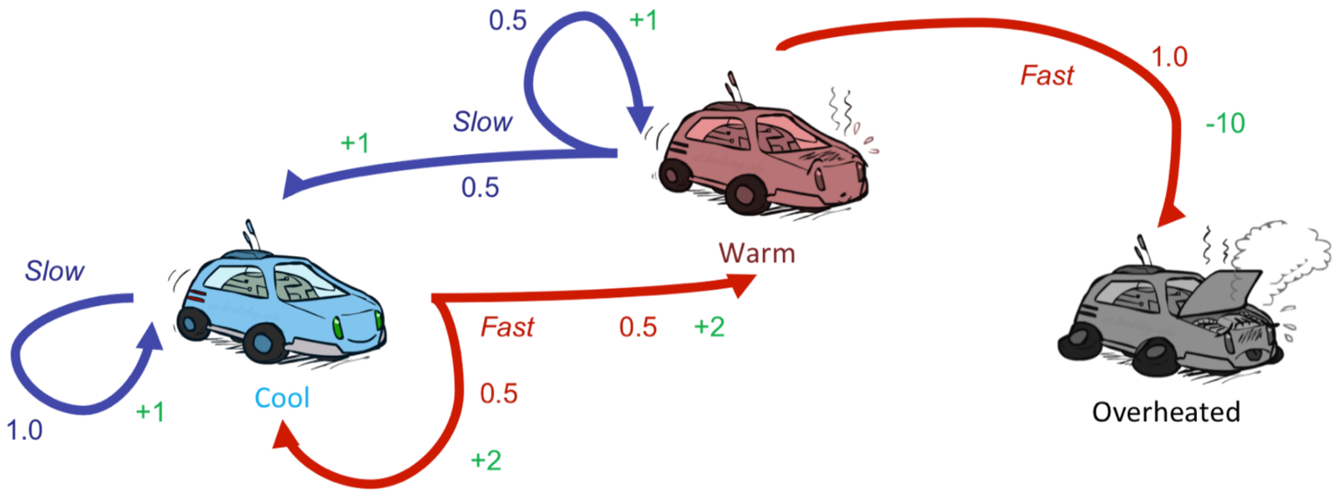
Hence, $V_2(warm) = 2.5$

$V_1(cool) = 2,\ \ V_1\ (warm) = 1,\ \ V_1(overheated) = 0$

- $V_2(overheated) = 0$

Example: Value Iteration

V_2	3.5	2.5	0
V_1	2	1	0
V_0	0	0	0



Assume no discount!

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

Convergence of Value Iteration

- **Theorem: will converge to unique optimal values**
- **Stopping Criterion**
- **Let the discount factor be γ**
- **If we want to achieve: $\max_s |V_{k+1}(s) - V^*(s)| < \epsilon$,**

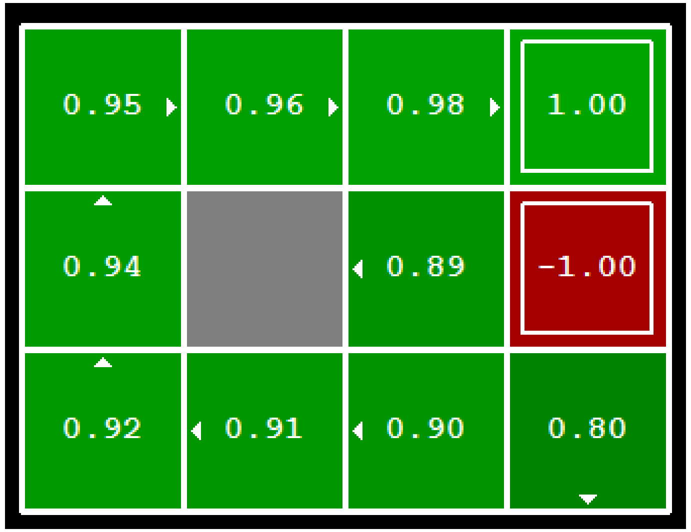
  then we need to run the iterations until

$$\max_s |V_{k+1}(s) - V_k(s)| < \epsilon(1-\gamma)/\gamma$$

Policy Extraction

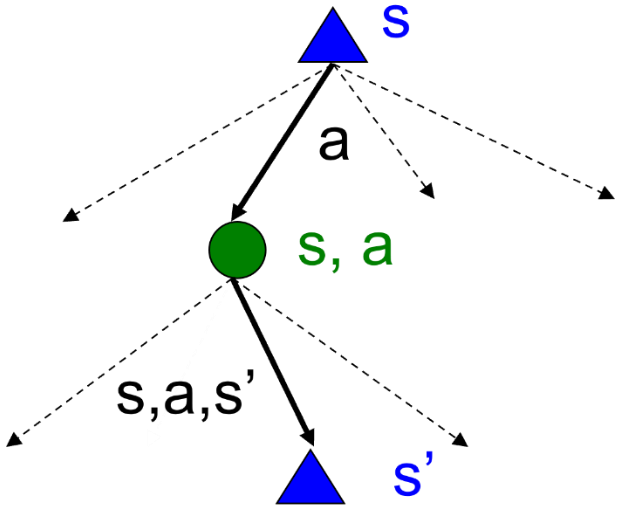
- **Assume we already calculated the optimal values V*(s)**
- **How to figure out the best action at state s?**
  - It's not obvious!

- **We need to do a mini-expectimax (one step look-ahead)**



- $\pi^*(s) = \arg\max_a Q(s,a)$

$$\pi^*(s) = \arg\max_a \sum_{s'} T(s,a,s') \times [R(s,a,s') + \gamma V^*(s')]$$

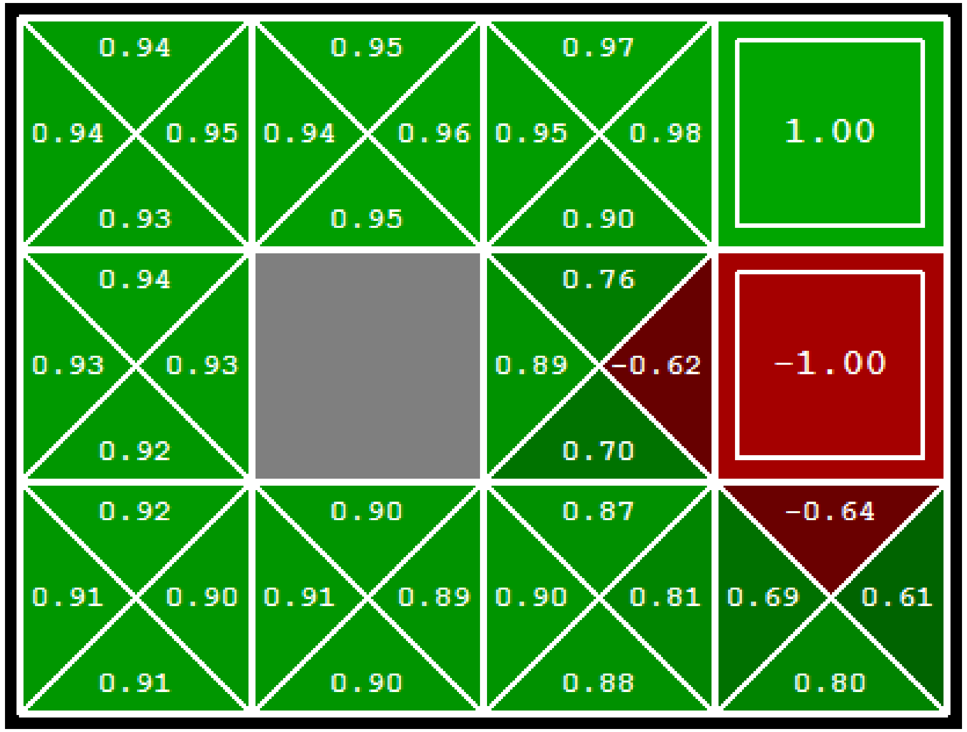


- **This is called policy extraction**
  - It gets the actions implied by the values

Computing Actions from Q-Values

- **Let's imagine we have the optimal q-values:**

$$Q^*(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right]$$



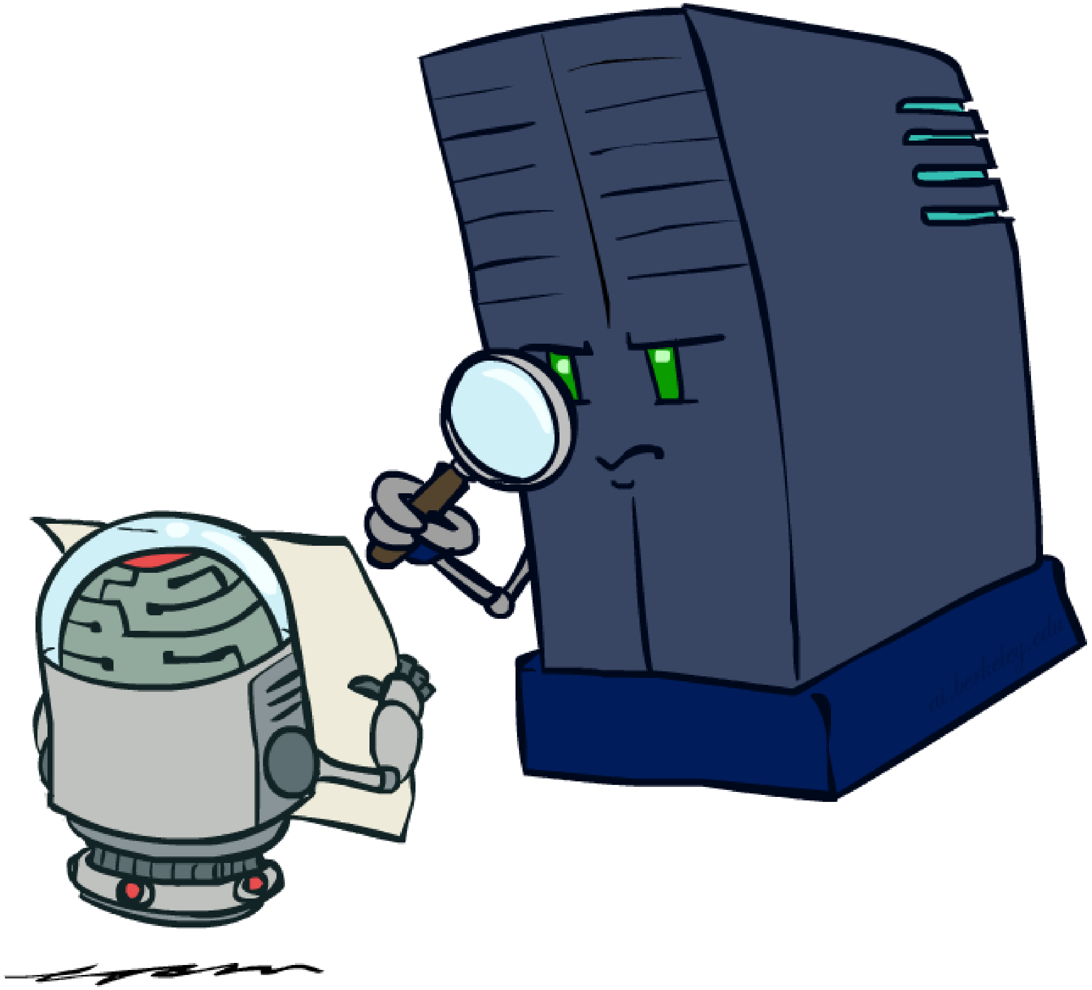
- **How to figure out the best action at state s?**
- **Completely trivial to decide!**

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

- **Important lesson: actions are easier to select from q-values than values!**

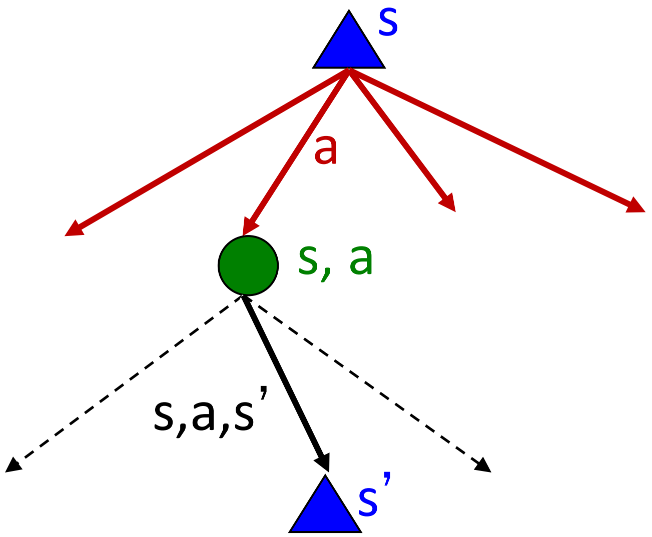
These slides were adapted from the MDP lecture slides of cs188 developed at UC Berkeley, http://ai.berkeley.edu.

Policy Evaluation

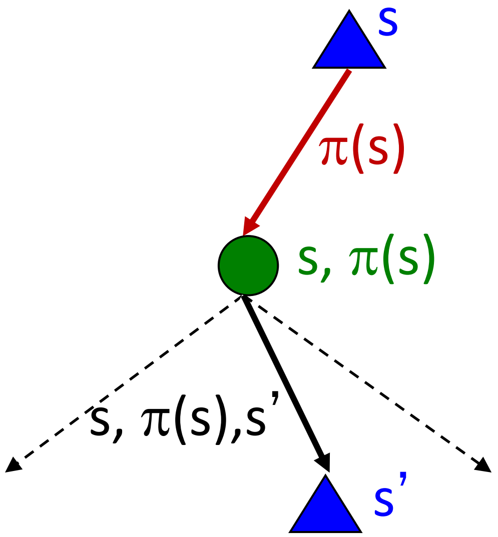


Fixed Policies

When no policy is told:
Do the optimal action



When a policy π is told:
Do what π says to do



- **When no policy is told: Expectimax trees max over all actions to compute the optimal values**
- **When a fixed policy π is told: the tree would be simpler - only one action per state**

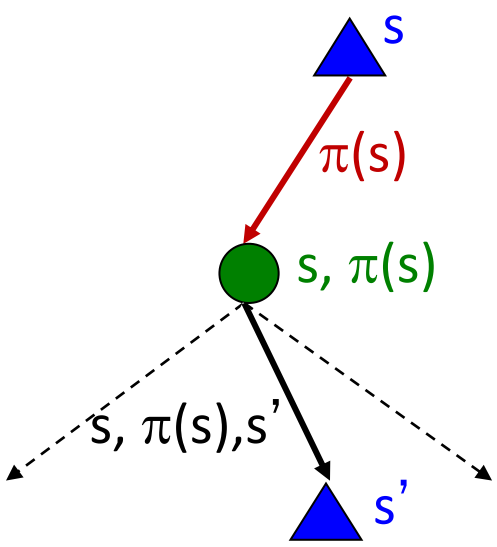
Utilities for a Fixed Policy

- **Define the value/utility of a state s, under a fixed policy π:**

  $V^{\pi}(s)$ = expected total discounted rewards starting in s and following π

- **Recursive relation**

$$V^{\pi}(s) = \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V^{\pi}(s')]$$

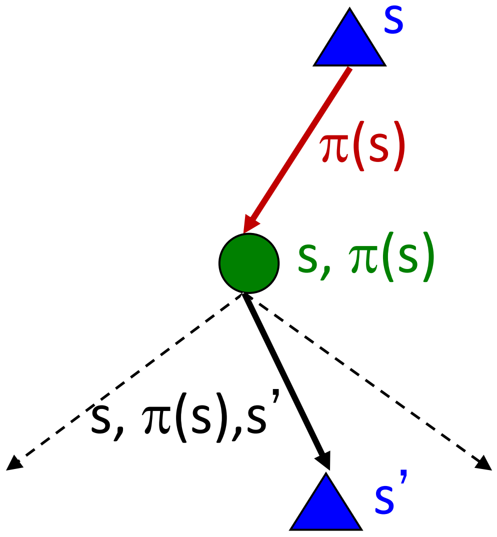


These slides were adapted from the MDP lecture slides of cs188 developed at UC Berkeley, http://ai.berkeley.edu.

Policy Evaluation

- **How do we calculate the values $V^{\pi}(s)$ for a fixed policy π?**

$$V^{\pi}(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^{\pi}(s')]$$



- **Idea 1: Turn recursive Bellman equations into updates (like value iteration)**

$$V_0^{\pi}(s) = 0$$

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

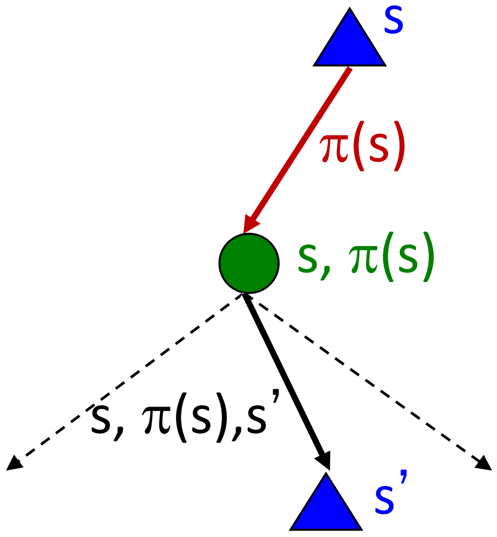
- **Complexity: $O(|S|^2)$ per iteration**

Policy Evaluation

- **How do we calculate the values $V^{\pi}(s)$ for a fixed policy π?**

$$V^{\pi}(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^{\pi}(s')]$$

- **Idea 2: Without the maxes, the Bellman equations are just a linear system**
  - Solve with Python (or your favorite linear system solver)



$$V^{\pi}(s_1) = f_1(V^{\pi}(s_1), V^{\pi}(s_2), \dots, V^{\pi}(s_N))$$
$$V^{\pi}(s_2) = f_2(V^{\pi}(s_1), V^{\pi}(s_2), \dots, V^{\pi}(s_N))$$
$$\vdots$$
$$V^{\pi}(s_N) = f_N(V^{\pi}(s_1), V^{\pi}(s_2), \dots, V^{\pi}(s_N))$$

Assume there are N states, then
we have N unknowns and N linear equations